# 18. INTERNATIONAL GEOMETRY SYMPOSIUM

JULY 12-13, 2021

# ABSTRACTS BOOK

IN HONOUR OF PROF.DR. SADIK KELEŞ

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# 18TH INTERNATIONAL GEOMETRY SYMPOSIUM

# ABSTRACTS BOOK

# Proceedings of the 18th International Geometry Symposium

Edited By:
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Prof.Dr. Erol KILIÇ
Prof.Dr. H. Bayram KARADAĞ
Prof.Dr. M. Kemal ÖZDEMÌR

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# Proceedings of the 18th International Geometry Symposium

July 12-13, 2021
Malatya, Turkey

Jointly Organized by
İnönü University

# TABLE OF CONTENTS

# Foreword

The 18th International Geometry Symposium has been planned to be held face-to-face in honor of Prof. Dr. Sadık KELEŞ on 01-04 July 2020; but due to the COVID-19 pandemic which affected the whole world, it has been postponed to be held online on July 12-13, 2021. We would like to thank Prof. Dr. Ahmet KIZILAY, the Rector of İnönü University and our honorary president of the symposium, who supported us for the 18th Geometry Symposium to be held at our university.

We would also like to thank our Head of Mathematics Department Prof. Dr. A. İhsan SİVRİDAĞ, the management and members of the Geometricians Association for their interest and support. Continuity in holding an international symposium is a team effort that requires dedication. For this reason, we would like to thank the symposium scientific and organizing committee members for all their support.

In addition, we would like to thank the invited speakers who participated in our symposium from Turkey and abroad, and all the participants who supported the symposium by participating in the symposium with and without papers. With the hope that the studies presented in this symposium are going to contribute to science and scientists...

Prof. Dr. Rıfat GÜNEŞ
İnönü University

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# Online Symposium Programme

# 12 July 2021 Monday (Time zone: UTC+3 / İstanbul)

| OPENING CEREMONY | |
|---|---|
| Please click here to join the Opening Ceremony | |
| Chair: Prof. Dr. Soley Ersoy | |
| **09:30-10:10** | Prof. Dr. Rıfat GÜNEŞ |
| | Prof. Dr. Ali İhsan SİVRİDAĞ |
| | Prof. Dr. Murat TOSUN (Sakarya University, Turkey) |
| | Prof. Dr. Sadık KELEŞ |
| | Prof. Dr. Ahmet KIZILAY (Rector of İnönü University) |

| PLENARY LECTURE | |
|---|---|
| Please click here to join the Plenary Lecture | |
| Chair: Prof. Dr. Kadri ASLAN | |
| **10:15-11:00** | **Invited Spiker: Prof. Dr. Bayram ŞAHİN**<br>**"Recent Developments on Conformal Submersions in Riemannian Geometry"** |

# 12 July 2021 Monday

## SESSION I / 11:05-12:05 (Time zone: UTC+3 / İstanbul)

| HALL 1 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Günay Öztürk | | Vice-Chair: Assoc. Prof. Dr. Betül Bulca |
| **Time** | **Author(s)** | **Title** |
| **11:05-11:20** | Kadri Arslan, Yılmaz Aydın, Betül Bulca | General rotational surfaces in Euclidean spaces |
| **11:20-11:35** | Günay Öztürk, İlim Kişi | On $L_1$-pointwise 1-type Gauss map of tubular surface in $G_3$ |
| **11:35-11:50** | Nurettin Cenk Turgay | A Note On the Surfaces in $\mathbb{E}^4$ with Generalized 1-Type Gauss Map |
| **11:50-12:05** | Semra Yurttançıkmaz | The Darboux Frame of Curves Lying On The Parallel-Like Surfaces in $\mathbb{E}^3$ |

| HALL 2 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Atakan Tuğkan Yakut | | Vice-Chair: Assoc. Prof. Dr. Zehra Özdemir |
| **Time** | **Author(s)** | **Title** |
| **11:05-11:20** | Ferhat Taş | Quaternion Vector Fields |
| **11:20-11:35** | Zehra Özdemir | Rigid motions of the polarization plane in the optical fiber through quaternion algebra |
| **11:35-11:50** | Kemal Eren, Soley Ersoy | Moving Quaternionic Curves and Modified Korteweg-de Vries Equation |
| **11:50-12:05** | Zülal Derin, Mehmet Ali Güngör | Electromagnetism and Maxwell's equations in terms of elliptic biquaternions in relativistic notation |

# 12 July 2021 Monday

## SESSION I / 11:05-12:05 (Time zone: UTC+3 / İstanbul)

| HALL 3 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Mahmut Ergüt | | Vice-Chair: Assoc. Prof. Dr. Sibel Pasalı Atmaca |
| **Time** | **Author(s)** | **Title** |
| **11:05-11:20** | Ömer Akgüller, Mehmet Ali Balcı, Sibel Pasalı Atmaca | Distribution of Discrete Geodesics on Point Set Surfaces |
| **11:20-11:35** | Abdussamet Çalışkan | The quaternionic ruled surfaces in terms of Bishop frame |
| **11:35-11:50** | Nazlı Yazıcı Gözütok | On the Regular Maps of Large Genus |
| **11:50-12:05** | Ayla Erdur Kara, Muhittin Evren Aydın, Mahmut Ergüt | Hypersurfaces with the lowest center of gravity in space forms |

| HALL 4 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Hakan Mete Taştan | | Vice-Chair: Assoc. Prof. Dr. Mustafa Kalafat |
| **Time** | **Author(s)** | **Title** |
| **11:05-11:20** | Hakan Mete Taştan, Sibel Gerdan Aydın | On Warped-Twisted Product Submanifolds |
| **11:20-11:35** | Mustafa Kalafat, Ramazan Sarı | On special submanifolds of the Page space |
| **11:35-11:50** | Ali Uçum | On k-type hyperbolic slant helices in 3-dimensional hyperbolic space |
| **11:50-12:05** | Eyüp Yalçınkaya | SU(3) Structure on Submanifolds of Locally Conformal Spin(7) Structure with 2-plane Field |

# 12 July 2021 Monday

## PLENARY LECTURE

Please click here to join the Plenary Lecture

Chair: Prof. Dr. Bayram ŞAHİN

| | |
|---|---|
| **13:00-13:45** | **Invited Spiker: Prof. Dr. Anna Maria Fino**<br>**"Closed $G_2-$structures and Laplacian flow"** |

## SESSION II / 13:50-14:50 (Time zone: UTC+3 / İstanbul)

### HALL 1

Please click here to join the Hall

Chair: Prof. Dr. Fatma Muazzez Şimşir    Vice-Chair: Assoc. Prof. Dr. Serhan Eker

| Time | Author(s) | Title |
|---|---|---|
| **13:50-14:05** | Okan Duman | Some Results on Projections of Affine Vector Fields on Homogeneous Spaces |
| **14:05-14:20** | Mustafa Kalafat, Özgür Kelekçi | Locally conformally flat metrics on surfaces of general type |
| **14:20-14:35** | Serhan Eker | Some Estimates in Terms of The Divergencefree Symmetric Tensor and It's Trace |
| **14:35-14:50** | Fatma Muazzez Şimşir | On translation-like covering transformations |

# 12 July 2021 Monday

## SESSION II / 13:50-14:50 (Time zone: UTC+3 / İstanbul)

| HALL 2 | Please click here to join the Hall | |
|---|---|---|
| **Chair: Prof. Dr. Arif Salimov** | | **Vice-Chair: Assoc. Prof. Dr. Filiz Ocak** |
| **Time** | **Author(s)** | **Title** |
| **13:50-14:05** | Filiz Ocak | Golden Structure on the Cotangent Bundle with Sasaki Type Metrics |
| **14:05-14:20** | Arif Salimov, Seher Aslancı, Fidan Jabrailzade | Problems of lifts concerning dual-holomorphic functions |
| **14:20-14:35** | Seher Aslancı, Tarana Sultanova | Some notes on deformed lifts |
| **15:40-15:55** | Çağatay Madan, Gülhan Ayar, Nesip Aktan | Nearly Cosymplectic Manifolds with Tanaka-Webster Connection |

| HALL 3 | Please click here to join the Hall | |
|---|---|---|
| **Chair: Prof. Dr. Sıddıka Özkaldı Karakuş** | | **Vice-Chair: Assoc. Prof. Dr. Önder Gökmen YILDIZ** |
| **Time** | **Author(s)** | **Title** |
| **13:50-14:05** | Bahar Doğan Yazıcı, Sıddıka Özkaldı Karakuş, Murat Tosun | On Framed Tzitzeica Curves in Euclidean Space |
| **14:05-14:20** | Mahmut Mak, Melek Demir | Some Special Legendre Mates of Spherical Legendre Curves |
| **14:20-14:35** | Uğur Gözütok, -Hüsnü Anıl Çoban, Yasemin Sağıroğlu | On the Projective Equivalence of Rational Algebraic Curves |
| **14:35-14:50** | Edanur Ergül, Salim Yüce | The Special Curves of Fibonacci and Lucas Curves |

# 12 July 2021 Monday

## SESSION II / 13:50-14:50 (Time zone: UTC+3 / İstanbul)

| HALL 4 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Nihal Özgür | | Vice-Chair: Assoc. Prof. Dr. Nihal Taş |
| **Time** | **Author(s)** | **Title** |
| **13:50-14:05** | Nihal Taş, Nihal Özgür | New results on "fixed-circle problem" |
| **14:05-14:20** | Nihal Özgür, Nihal Taş | On the geometry of $\varphi$-fixed points |
| **14:20-14:35** | Hülya Aytimur, Nihal Taş | Some geometric results on $S_b$-metric spaces |
| **14:35-14:50** | Murat Candan | Some matrix transformations related to new specified spaces |

## SESSION III / 14:55-15:55 (Time zone: UTC+3 / İstanbul)

| HALL 1 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Esmaeil Peyghan | | Vice-Chair: Assoc. Prof. Dr. Shahid Ali |
| **Time** | **Author(s)** | **Title** |
| **14:55-15:10** | Miroslava Antić, Luc Vrancken | Warped product, minimal, conformally flat, Lagrangian submanifolds in complex space forms |
| **15:10-15:25** | Esmaeil Peyghan | Geometry structures on Hom-Lie groups and Hom-Lie algebras |
| **15:25-15:40** | Habil Fattayev | On a new class of Riemannian metrics on the coframe bundle |
| **15:40-15:55** | Tanveer Fatima, Shahid Ali | Submersion of CR-Warped Product Submanifold of a Nearly Kaehler Manifold |

# 12 July 2021 Monday

## SESSION III / 14:55-15:55 (Time zone: UTC+3 / İstanbul)

| HALL 2 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Kazım İlarslan | | Vice-Chair: Assoc. Prof. Dr. Çetin Camcı |
| **Time** | **Author(s)** | **Title** |
| **14:55-15:10** | Hatice Altın Erdem, Kazım İlarslan | New Resulst For Spacelike Bertrand Curves In Minkowski 3-Space |
| **15:10-15:25** | Çetin Camcı, Ali Uçum, Kazım İlarslan | New results for curve pairs in Euclidean 3-space |
| **15:25-15:40** | Özcan Bektaş, Zafer Bekiryazıcı | A new type of osculating curve in $E^n$ |
| **15:40-15:55** | Fatma Gökcek, Ali Uçum, Kazım İlarslan | New Results for Cartan Null Bertrand Curves in Minkowski 3-Space |

| HALL 3 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Mustafa Çalışkan | | Vice-Chair: Assist. Prof. Dr. Süleyman Şenyurt |
| **Time** | **Author(s)** | **Title** |
| **14:55-15:10** | Emel Karaca, Mustafa Çalışkan | A new perspective for the intersection of two ruled surfaces |
| **15:10-15:25** | Yunus Öztemir, Mustafa Çalışkan | On the intersection curve of two ruled surfaces in dual space |
| **15:25-15:40** | Burak Şahiner | Some Notes on Ruled Surfaces according to Alternative Moving Frame in Euclidean 3-space |
| **15:40-15:55** | Davut Canlı, Süleyman Şenyurt, Kebire Hilal Ayvacı | On the ruled surfaces generated by Sannia Frame based on alternative frame |

# 12 July 2021 Monday

## SESSION III / 14:55-15:55 (Time zone: UTC+3 / İstanbul)

| HALL 4 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Salim Yüce | | Vice-Chair: Assoc. Prof. Dr. Şaban Güvenç |
| **Time** | **Author(s)** | **Title** |
| **14:55-15:10** | Hülya Aytimur | Chen-Ricci Inequalities for Anti-Invariant Riemanian Submersions From Cosymplectic Space Forms |
| **15:10-15:25** | İpek Ebru Karaçay, Salim Yüce | A new approach to generalized cantor set for $\mathbb{R}^2$ in fractal geometry |
| **15:25-15:40** | Arzu Cihan, Mehmet Ali Güngör | A Study on Commutative Elliptic Octonion Matrices |
| **15:40-15:55** | Erdem Kocakuşaklı | Graphs With Density |

## SESSION IV / 16:00-17:00 (Time zone: UTC+3 / İstanbul)

| HALL 1 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Erol Yaşar | | Vice-Chair: Assoc. Prof. Dr. Mehmet Gülbahar |
| **Time** | **Author(s)** | **Title** |
| **16:00-16:15** | İdris Ören, Gayrat Beshimov, Djavvat Khadjiev | Euclidean invariants of plane paths |
| **16:15-16:30** | Gayrat Beshimov, İdris Ören, Djavvat Khadjiev | The concept of the notion of a figure in two-dimensional Euclidean geometry and its Euclidean invariants |
| **16:30-16:45** | Sema Kazan, Cumali Yıldırım | Screen Almost Semi-Invariant Lightlike Submanifolds of indefinite Kaehler Manifolds |
| **16:45-17:00** | Erol Kılıç, Mehmet Gülbahar, Ecem Kavuk, Sadık Keleş | Some notes on Ricci soliton lightlike hypersurfaces admitting concurrent vector fields |

# 12 July 2021 Monday

## SESSION IV / 16:00-17:00 (Time zone: UTC+3 / İstanbul)

| HALL 2 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Mehmet Bektaş | | Vice-Chair: Assoc. Prof. Dr. Bilal Eftal Acet |
| **Time** | **Author(s)** | **Title** |
| **16:00-16:15** | Müslüm Aykut Akgün, Bilal Eftal Acet | Sesqui Harmonic Curves in LP-Sasakian Manifolds |
| **16:15-16:30** | Müslüm Aykut Akgün, Ali Ihsan Sivridağ | Curves on Lorentzian Manifolds |
| **16:30-16:45** | Ramazan Sarı | Generic $\xi^{\perp}-$Riemannian Submersions from Sasakian Manifolds |
| **16:45-17:00** | Ahmet Mollaoğulları, Çetin Camcı | $\alpha$- Sasakian structure on product of a Kahler manifold and an open curve |

| HALL 3 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Mustafa Düldül | | Vice-Chair: Assist. Prof. Dr. Mustafa Altın |
| **Time** | **Author(s)** | **Title** |
| **16:00-16:15** | Bedia Merih Özçetin, Mustafa Düldül | On the intersection curve of implicit hypersurfaces in $E^n$ |
| **16:15-16:30** | Esra Erdem, Münevver Yıldırım Yılmaz | Special Characterizations for Normal Curves According to Type-2 Quaternionic Frame in $\mathbb{R}^4$ |
| **16:30-16:45** | Gülden Altay Suroğlu | Dual Representation of Ribbon Surface |
| **16:45-17:00** | Ahmet Kazan, Mustafa Altın | Parametric Expressions of Rotational Hypersurfaces According to Curvatures in $E_1^4$ |

# 12 July 2021 Monday

## SESSION IV / 16:00-17:00 (Time zone: UTC+3 / İstanbul)

| HALL 4 | Please click here to join the Hall | |
|---|---|---|
| **Chair: Prof. Dr. Nesip Aktan** | | **Vice-Chair: Assoc. Prof. Dr. Tunahan Turhan** |
| **Time** | **Author(s)** | **Title** |
| **16:00-16:15** | Gizem Köprülü, Bayram Şahin | Biharmonic Curves along Riemannian Submersions and Riemannian Maps |
| **16:15-16:30** | Hatice Kübra Konak, Mert Taşdemir, Bayram Şahin | Certain curves on Riemannian manifolds |
| **16:30-16:45** | Tunahan Turhan, Gözde Özkan Tükel, Bayram Şahin | On the Characterization of a Riemannian map by Hyperelastic Curves |
| **16:45-17:00** | Gözde Özkan Tükel, Bayram Şahin, Tunahan Turhan | On the Geometry of a Riemannian map with Helices |

# 12 July 2021 Monday

## SESSION V / 17:05-18:05 (Time zone: UTC+3 / İstanbul)

| HALL 1 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Bülent Karakaş | | Vice-Chair: Assoc. Prof. Dr. Yasemin Soylu |
| **Time** | **Author(s)** | **Title** |
| **17:05-17:20** | Şeyda Kılıçoğlu, Süleyman Şenyurt | The area of the Bézier polygonal region contains the Bezier Curve and derivatives in $E^3$ |
| **17:20-17:35** | Şeyda Kılıçoğlu, Süleyman Şenyurt | On the Bertrand mate of a cubic Bézier curve by using matrix representation in $E^3$ |
| **17:35-17:50** | Hakan Gündüz, Müge Karadağ, H. Bayram Karadağ | Generalized Trigonometric B-Spline and Nurbs Curves and Surfaces with shape parameters |
| **17:50-18:05** | Bülent Karakaş, Şenay Baydaş | Two Different Models for Spatial Boomerang Motion |

| HALL 2 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Cihan Özgür | | Vice-Chair: Assoc. Prof. Dr. Mehmet Akif Akyol |
| **Time** | **Author(s)** | **Title** |
| **17:05-17:20** | Mehmet Akif Akyol, Tanveer Fatima | Almost Yamabe Solitons and Torqued Vector Fields on a Total Manifold of Almost Hermitian Submersions |
| **17:20-17:35** | Feyza Esra Erdoğan, Bayram Şahin, Rıfat Güneş | Study of Isotropic Riemannian Submersions |
| **17:35-17:50** | Tuna Bayrakdar, Zahide Ok Bayrakdar | The geometry of a surface in the Riemannian manifold associated with simple harmonic oscillator |
| **17:50-18:05** | Sinem Güler, Hakan Mete Taştan | Some Soliton Structures on Twisted Product Manifolds |

# 12 July 2021 Monday

## SESSION V / 17:05-18:05 (Time zone: UTC+3 / İstanbul)

### HALL 3

Please click here to join the Hall

Zoom Video Conferencing

Chair: Prof. Dr. Nuri Kuruoğlu — Vice-Chair: Assoc. Prof. Dr. Tevfik Şahin

| Time | Author(s) | Title |
|---|---|---|
| 17:05-17:20 | Önder Gökmen Yıldız, Ebru Gürsaç | A note on involute-evolute curves of framed curves in the Euclidean Space |
| 17:20-17:35 | Gülnur Özyurt, Tevfik Şahin | Position Vectors of curves in the Isotropic Space $I^3$ |
| 17:35-17:50 | Nurcan Demircan Bekar, Ömer Pekşen | Types and Invariant Parametrizations of Regular and d-Regular Curves |
| 17:50-18:05 | Şaban Güvenç | On curves satisfying the Lorentz Equation in S-manifolds endowed with a particular affine metric connection |

### HALL 4

Please click here to join the Hall

Zoom Video Conferencing

Chair: Prof. Dr. Murat Kemal Karacan — Vice-Chair: Assoc. Prof. Dr. Ali Hikmet Değer

| Time | Author(s) | Title |
|---|---|---|
| 17:05-17:20 | Melek Erdoğdu, Ayşe Yavuz | On soliton surface associated with Nonlinear Schrödinger (NLS) equation |
| 17:20-17:35 | Ayşe Yavuz, Melek Erdoğdu | Position vector of spacelike curves by a different approach |
| 17:35-17:50 | Ümmügülsün Akbaba, Tuğba Tuylu, Ali Hikmet Değer | Images of Some Discs Under the Linear Fractional Transformation of Special Continued Fractions |
| 17:50-18:05 | Tuğba Tuylu, Ümmügülsün Akbaba, Ali Hikmet Değer | Images of Minimal-Length Hyperbolic Paths on the Poincare Disc |

# 13 July 2021 Tuesday (Time zone: UTC+3 / İstanbul)

| PLENARY LECTURE | |
|---|---|
| Please click here to join the Plenary Lecture | |
| Chair: Prof. Dr. Selcen Yüksel Perktaş | |
| **09:30-10:15** | **Invited Spiker: Prof. Dr. Jeong Hyeong Park**<br>**"Einstein hypersurfaces in harmonic spaces"** |

# 13 July 2021 Tuesday

## SESSION VI / 10:20-11:20 (Time zone: UTC+3 / İstanbul)

| HALL 1 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. İsmail Gök | | Vice-Chair: Assoc. Prof. Dr. Zehra Özdemir |
| **Time** | **Author(s)** | **Title** |
| **10:20-10:35** | Kübra Şahin, Zehra Özdemir | The charged point-particle trajectories on timelike surfaces |
| **10:35-10:50** | Filiz Ertem Kaya | A Survey for Evolute-Involute Partner Curves in the Euclidean 3-Space |
| **10:50-11:05** | Süleyman Şenyurt, Osman Çakır | Characterizations of a Bertrand Curve According to Darboux Vector |
| **11:05-11:20** | Hazal Ceyhan, Zehra Özdemir, İsmail Gök, F. Nejat Ekmekci | Electromagnetic curves through Alternative Moving (N,C,W) Frame |

# 13 July 2021 Tuesday

## SESSION VI / 10:20-11:20 (Time zone: UTC+3 / İstanbul)

### HALL 2 — Please click here to join the Hall

Chair: Prof. Dr. Mihriban Alyamaç Külahcı — Vice-Chair: Assoc. Prof. Dr. Fatma Karaca

| Time | Author(s) | Title |
|---|---|---|
| 10:20-10:35 | Şener Yanan | New kinds of conformal Riemannian maps |
| 10:35-10:50 | Sadık Keleş, Vildan Ayhan, Selcen Yüksel Perktaş | Slant Submanifolds of Almost Poly-Norden Metric Manifolds |
| 10:50-11:05 | Mustafa Gök, Erol Kılıç, Sadık Keleş | Some remarks on invariant and anti-invariant submanifolds of a golden Riemannian manifold |
| 11:05-11:20 | Fatma Karaca | On some properties of gradient Ricci-Yamabe solitons on warped product manifolds |

### HALL 3 — Please click here to join the Hall

Chair: Prof. Dr. Ömer Pekşen — Vice-Chair: Assoc. Prof. Dr. Müge Karadağ

| Time | Author(s) | Title |
|---|---|---|
| 10:20-10:35 | Anıl Altınkaya, Mustafa Çalışkan | On the Ruled Surfaces of the B-Lift Curves |
| 10:35-10:50 | Fatma Ateş | Ruled surface generated by constant slope direction vector in Galilean 3-space |
| 10:50-11:05 | Gülşah Aydın Şekerci | Pedals and primitivoids of frontals in Minkowski plane |
| 11:05-11:20 | Serkan Çelik, Hatice Kuşak Samancı, H. Bayram Karadağ, Sadık Keleş | Conchoidal Twisted Surfaces in Euclidean 3-Space |

# 13 July 2021 Tuesday

## SESSION VII / 11:25-12:25 (Time zone: UTC+3 / İstanbul)

| HALL 1 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Mukut Mani Tripathi | | Vice-Chair: Assist. Prof. Dr. Şemsi Eken Meriç |
| **Time** | **Author(s)** | **Title** |
| **11:25-11:40** | Jay Prakash Singh, Mohan Khatri | On semiconformal curvature tensor in $(\kappa, \mu)$-contact metric manifold |
| **11:40-11:55** | Jay Prakash Singh, Mohan Khatri | On almost pseudo semiconformally symmetric manifold |
| **11:55-12:10** | Mukut Mani Tripathi | Slant Submanifolds of Conformal Sasakian Space Forms |
| **12:10-12:25** | Cornelia-Livia-Bejan, Erol Yaşar, Şemsi Eken Meriç | Contact-Complex Riemannian Submersions and $\eta$-Ricci Solitons |

## SESSION VII / 11:25-12:25 (Time zone: UTC+3 / İstanbul)

| HALL 2 | Please click here to join the Hall | |
|---|---|---|
| Chair: Assoc. Prof. Dr. Sezai Kızıltuğ | | Vice-Chair: Assoc. Prof. Dr. Ali Çakmak |
| **Time** | **Author(s)** | **Title** |
| **11:25-11:40** | Doğuş İlgen, Sıddıka Özkaldı Karakuş | Dual Quaternions and Translational Surfaces |
| **11:40-11:55** | Sezai Kızıltuğ, Ali Çakmak, Gökhan Mumcu | On Tubular Surfaces with Modified Frame in 3-Dimensional Galilean Space |
| **11:55-12:10** | Fatma Almaz, Mihriban Alyamaç Külahcı | The Geodesıcs on Specıal Tubular Surfaces Generated by Darboux Frame ın $G_3$ |
| **12:10-12:25** | Fatma Almaz, Mihriban Alyamaç Külahcı | The Geometrıcal Interpretatıon of The Energy ın The Null Cone $Q^2$ |

# 13 July 2021 Tuesday

## SESSION VII / 11:25-12:25 (Time zone: UTC+3 / İstanbul)

| HALL 3 | Please click here to join the Hall | |
|---|---|---|
| **Chair: Prof. Dr. F. Nejat Ekmekci** | | **Vice-Chair: Assoc. Prof. Dr. Ümit Yıldırım** |
| **Time** | **Author(s)** | **Title** |
| **11:25-11:40** | Pelin Özlem Toy, Efe Dölek, Esat Avcı | Obtaining general terms of polygonal number sequences with areas of unit squares and area formula of right triangle |
| **11:40-11:55** | Mehmet Arslan, Ahmet Enis Guven | Polygonal Structure Analysis on the Poincare Disk Model |
| **11:55-12:10** | Oğuzhan Bahadır, Hande Türkmençalıkoğlu | Looking at the Concept of Entropy from Information Theory |
| **12:10-12:25** | Gülsüm Yeliz Şentürk, Nurten Gürses, Salim Yüce | A New Look on Oresme Numbers: Dual-Generalized Complex Component Extension |

# 13 July 2021 Tuesday (Time zone: UTC+3 / İstanbul)

## PLENARY LECTURE

Please click here to join the Plenary Lecture

| Chair: Assoc. Prof. Dr. Muhittin Evren AYDIN | |
|---|---|
| **13:00-13:45** | **Invited Spiker: Prof. Dr. Gabriel Eduard Vilcu**<br>**"Curvature invariants, optimal inequalities and ideal submanifolds in space forms"** |

# 13 July 2021 Tuesday

## SESSION VIII / 13:50-14:50 (Time zone: UTC+3 / İstanbul)

### HALL 1 — Please click here to join the Hall

**Chair: Prof. Dr. Anna Maria Fino** — **Vice-Chair: Assist. Prof. Dr. İnan Ünal**

| Time | Author(s) | Title |
|---|---|---|
| **13:50-14:05** | Arfah Arfah, Gül Tuğ | $\eta$-Ricci solitons on lightlike hypersurfaces |
| **14:05-14:20** | Arfah Arfah | Geometry of Kahler manifold endowed with symmetric non-metric connection |
| **14:20-14:35** | Ajit Barman, İnan Ünal | Z-tensor on Kenmotsu manifolds |
| **14:35-14:50** | M. R. Amruthalakshmi, D. G. Prakasha, İnan Ünal | Some results on $\alpha$-cosymplectic manifolds |

### HALL 4 — Please click here to join the Hall

**Chair: Prof. Dr. Münevver Yıldırım Yılmaz** — **Vice-Chair: Assoc. Prof. Dr. Semra Kaya Nurkan**

| Time | Author(s) | Title |
|---|---|---|
| **13:50-14:05** | Süleyman Şenyurt, Kebire Hilal Ayvacı, Davut Canlı | Some characterizations of spherical indicatrix curves generated by Flc frame |
| **14:05-14:20** | Çetin Camcı, Mehmet Gümüş, Ahmet Mollaoğulları, Kazım İlarslan | Chracterization of PH-Helical curves in Euclidean 4-space |
| **14:20-14:35** | Semra Kaya Nurkan | Integral Curves of Special Smarandache Curves |
| **14:35-14:50** | Aykut Has, Beyhan Yılmaz | Conformable Specıal Curves Accordıng to Conformable Frame ın Euclıdean 3-Space |

## SESSION VIII / 13:50-14:50 (Time zone: UTC+3 / İstanbul)

| HALL 3 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Mehmet Baran | | Vice-Chair: Assoc. Prof. Dr. Zühal Küçükarslan Yüzbaşı |
| **Time** | **Author(s)** | **Title** |
| **13:50-14:05** | Benen Akıncı, Hasan Altınbaş, Levent Kula | Timelike pythagorean normal surfaces with normal $N{=}e_3$ in Minkowski space |
| **14:05-14:20** | Çiğdem Turan, Mustafa Altın, H. Bayram Karadağ, Sadık Keleş | Parametric Representation of Hypersurfaces Pencil with Common Geodesic in $E_1^4$ |
| **14:20-14:35** | Mehmet Bektaş, Zühal Küçükarslan Yüzbaşı | Surfaces with a Common Asymptotic Curve in terms of Alternative Moving Frame in Lie Group |
| **14:35-14:50** | Erdoğan Zengin, Mehmet Baran | $T_1$ Limit Spaces |

# 13 July 2021 Tuesday

## SESSION IX / 14:55-15:55 (Time zone: UTC+3 / İstanbul)

| HALL 1 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Mehmet Atçeken | | Vice-Chair: Assoc. Prof. Dr. Feyza Esra Erdoğan |
| **Time** | **Author(s)** | **Title** |
| **14:55-15:10** | Semra Zeren, Ahmet Yıldız | Semi-slant Submanifolds of Kenmotsu Manifold with respect to the Schouten-van Kampen Connection |
| **15:10-15:25** | Şerife Nur Bozdağ, Feyza Esra Erdoğan, Selcen Yüksel Perktaş | On f-Biharmonic and f-Biminimal Curves in Kenmotsu Manifolds |
| **15:25-15:40** | Mehmet Atçeken, Ümit Yıldırım | Ricci Solitons on Ricci Pseudosymmetric an Almost Kenmotsu $(\kappa, \mu, \upsilon)$-Space |

# 13 July 2021 Tuesday

## SESSION IX / 14:55-15:55 (Time zone: UTC+3 / İstanbul)

| HALL 2 | Please click here to join the Hall | |
|---|---|---|
| Chair: Prof. Dr. Levent Kula | | Vice-Chair: Assoc. Prof. Dr. Melek Erdoğdu |
| **Time** | **Author(s)** | **Title** |
| **14:55-15:10** | Ayşe Yavuz, Melek Erdoğdu | Characterization of timelike Bertrand curve mate by means of differential equations for position vector |
| **15:10-15:25** | Ayşe Yavuz, Melek Erdoğdu | Characterization of spacelike Bertrand curve mate by using position vector |
| **15:25-15:40** | Hasan Altınbaş, Bülent Altunkaya, Levent Kula | Spacelike and timelike polynomial helices in the semi-Euclidean space $E_2^n$ |
| **15:40-15:55** | Cornelia Livia Bejan, Cem Sayar | Generic Submanifolds of Almost Contact Metric Manifolds |

# 13 July 2021 Monday (Time zone: UTC+3 / İstanbul)

## CLOSING CEREMONY

| Please click here to join the Closing Ceremony | |
|---|---|
| Chair: Prof. Dr. Soley Ersoy | |
| **16:00-** | The feelings and thoughts of the participants about the symposium and their expectations for the future |

## Presentation Notice for authors and Section Chairs/Moderators

**REMARKS :**

1. The authors must be ready in the meeting room at least 5 minutes prior to the start of the session. Presenters must introduce themselves to the session chair(s) and upload their Oral presentations to the computer.
2. Authors must be able to present on any day of the symposium the program cannot be tailored around specific requests from individual authors to present on particular days.
3. The international research symposium program is designed for original research contributions and presentations in all research fields. Presentations scheduled in the Oral sessions are drawn from a selection of the peer reviewed papers from a wide range of scientific and other disciplines of inquiry.

## ROLE OF THE SESSION CHAIR/MODERATOR

### The duties of the Session Chair include the following:

1. Arrive at the conference hall at least 5 minutes before the session begins. Identify the paper presenters and discussant(s) in advance, and introduce yourself. Remind each presenter of the time limits that apply, and describe the method you will use to alert them of time limits during the actual presentation.
2. At the start of the session, introduce yourself to the audience, announce the session/title, and offer a brief overview indicating how the papers are related.
3. Prior to each presentation, introduce the speaker, announce the paper's title, the name(s) of the author(s), and provide brief comments regarding the affiliation and/or background of each presenter. Identify the individual who will be speaking if it is someone other than the first author.
4. During the presentations enforce time limits strictly so that no author (or audience member) monopolizes someone else's time. Oral paper presentations each have 15 minutes (10 minutes for full presentation papers, 5 minutes for discussions).
5. Once presentations are complete, the remaining time can be used for informal discussion between the audience and session participants. It is your job to field questions from the audience.

# Invited Speakers

# Recent Developments on Conformal Submersions in Riemannian Geometry

**Bayram Şahin**

*Department of Mathematics,Faculty of Science, Ege University,35100 İzmir, TURKEY,*
*bayram.sahin@ege.edu.tr*

**Abstract**

The theory of Riemannian submersions is a new research area when compared to its immersion counterpart, Riemannian submanifolds. It was first studied in the literature by O'Neill [14] and Gray [10] in the mid-1960s.

After Watson's study [26] on the notion of holomorphic submersion, this research area has been active research area in manifold theory. The theory of Riemannian submersions has been large sections of Besse's book [1], Yano-Kon's book [28], and the updated research outcomes of this research area was presented in details in Ianus, Falcitelli and Pastore's book [9]. Conformal maps in mathematics have been the dominant factor in many research areas as they provide the opportunity to transform geometric objects by deforming them. Especially in the theory of harmonic morphisms [8], conformal maps appear as important and useful geometric notions to characterize harmonic morphism between Riemannian manifolds. In this talk, conformal Riemannian submersions will be considered and current studies on conformal Riemannian submersions defined on an almost Hermitian manifold will be presented.

## References

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# Curvature invariants, optimal inequalities and ideal submanifolds in space forms

**Gabriel Eduard Vilcu**

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**Abstract**

We derive some inequalities involving basic intrinsic and extrinsic invariants of submanifolds in several space forms. We provide examples showing that these inequalities are the best possible and obtain some classification results for ideal submanifolds (in the sense of B.-Y. Chen).

# Einstein hypersurfaces in harmonic spaces

**JeongHyeong Park, Yuri Nikolayevsky and Sinhwi Kim**

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*Department of Mathematics, Sungkyunkwan University, Suwon, 16419, Korea,* kimsinhwi@skku.edu

**Abstract**

Einstein hypersurfaces in Riemannian manifolds are very rare. Existence of an Einstein hypersurface imposes strong conditions on the ambient Riemannian manifold and it is reasonable to ask which (say homogeneous) spaces admit an Einstein hypersurface.

In this talk, we review the geometry of harmonic spaces, in particular, rank-one symmetric spaces. We present the classification of Einstein hypersurfaces in the Cayley projective plane and in its noncompact dual. This result completes the classification of Einstein hypersurfaces in rank-one symmetric spaces. Finally, we discuss the case of non-symmetric harmonic spaces.

# Closed $G_2-$structures and Laplacian flow

**Anna Maria Fino**

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**Abstract**

$G_2-$structures on $7-$manifolds are defined by a closed positive $3-$forms and constitute the starting point in various known and potential methods to obtain holonomy $G_2-$metrics. Although linear, the closed condition for a $G_2-$structure is very restrictive, and no general results on the existence of closed $G_2-$structures on compact 7-manifolds are known. In the seminar I will review known examples of compact $7-$manifolds admitting a closed $G_2-$structure. Moreover, I will discuss some results on the behaviour of the Laplacian $G_2-$flow starting from a closed $G_2-$structure whose induced metric satisfies suitable extra conditions.

# Abstracts of Oral Presentations

# Warped product, minimal, conformally flat, Lagrangian submanifolds in complex space forms

**Miroslava Antić, Luc Vrancken**

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*Laboratoire de Mathématiques pour lâĂŽIngénieur, Université Polytechnique Hauts-de-France, Valenciennes, France,* *luc.vrancken@uphf.fr, and*
*Department of Mathematics, KU Leuven, Leuven, Belgium,* *luc.vrancken@kuleuven.be*

**Abstract**

Submanifold $M^n$ of a complex space form $\widetilde{M}^n(4c)$ is Lagrangian if the corresponding almost complex structure $J$ maps the tangent space $TM_p^n$ into the normal space $NM_p^n$, for any point $p \in M^n$. If $n > 3$, the submanifold is conformally flat if and only if the Weyl tensor vanishes and then the corresponding Schouten tensor is of Codazzi type.

We investigate minimal, conformally flat, Lagrangian submanifolds of complex space forms, with $n \geq 4$ in terms of the eigenvalues of its Schouten tensor. We show that submanifolds with Schouten tensor admitting at most one eigenvalue of multiplicity one are either of constant sectional curvature or a warped product of an interval and a constant sectional curvature manifold and obtain their classification.

**Keywords**: Lagrangian submanifolds, complex space forms, comformally flat submanifolds.
**2010 Mathematics Subject Classification**: 53B25, 53B20.

# Geometry structures on Hom-Lie groups and Hom-Lie algebras

Esmaeil Peyghan

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*e-peyghan@araku.ac.ir*

**Abstract**

We describe two geometric notions, holomorphic Norden structures and Kähler-Norden structures on Hom-Lie groups, and prove a relationship between them on Hom-Lie groups in the left invariant setting. We study Kähler-Norden structures with abelian complex structures and give the curvature properties of holomorphic Norden structures on Hom-Lie groups. We show that any left-invariant holomorphic Hom-Lie group is a flat (holomorphic Norden Hom-Lie algebra carries a Hom-Left-symmetric algebra ) if its left-invariant complex structure (complex structure) is abelian. Also, we consider Hom-Lie groups and introduce left invariant almost contact structures on them (almost contact Hom-Lie algebras). On such Hom-Lie groups, we construct the almost contact metrics and the contact forms.

**Keywords**: Hom-Lie group, Hom-Lie algebra, Kähler-Norden structures, almost contact structures.
**2010 Mathematics Subject Classification**: 53C15, 53D05.

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# Quaternion Vector Fields

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**Abstract**

In this presentation, we show that any quaternion curve (surface) can be derived from curves. In addition, rotation of any vector in space with respect to these quaternion curves (surfaces) results in the curves (surfaces). Some examples are given and illustrated.

**Keywords**: Quaternions, rotations, curves, dual quaternions.
**2010 Mathematics Subject Classification**: 11R52, 15A66.

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# Distribution of Discrete Geodesics on Point Set Surfaces

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**Abstract**

Point set surfaces are one the most basic respresentation 3 dimensional Euclidean embedded data sets. Given the growing popularity and wide range of applications of this data source, it is very important to work directly with this representation without having to go through the intermediate step that can add computational complexity and surface fitting errors. Another important area where point sets are frequently used is the representation of high dimensional manifolds. This kind of high-dimensional and general isomorphic data is seen in almost all disciplines, from computational biology to image analysis and financial data. In this case, due to the high dimensionality, manifold reconstruction is impossible and the corresponding calculations have to be performed directly on the raw data. In this study, we give a novel aproach to determine distributions of geodesics on point set surfaces by using embedded graphs and their shortest paths. Our method is performed on several 3D discrete models and it is shown that such distributions are useful for similarity based problems.

**Keywords**: Point set sutface, discrete manifolds, geometric graphs, shortest path
**2010 Mathematics Subject Classification**: 52C05, 58J50, 68U05

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# Some notes on Ricci soliton lightlike hypersurfaces admitting concurrent vector fields

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**Abstract**

Ricci soliton lightlike hypersurfaces of a Lorentzian manifold are examined. An example of this frame of hypersurfaces is presented. With the help of some properties of concurrent vector fields, some characterizations are obtained.

**Keywords**: Ricci soliton, lightlike hypersurface, Lorentzian manifold.
**2010 Mathematics Subject Classification**: 53C25, 53C42, 53C50.

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# Some Results on Projections of Affine Vector Fields on Homogeneous Spaces

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**Abstract**

A control system on a connected Lie group

$$\dot{q} = \mathcal{X}_q + \sum_{i=1}^{n} u_i Y_q^i \tag{1}$$

is called to be linear if drift vector field $\mathcal{X}$ is linear which means that the flow is a 1-parameter group of automorphisms and $Y^i$'s are right invariant vector fields. When a right invariant vector field is added to drift vector field $\mathcal{X}$, the vector field obtained is called affine. The system (1) defined on a manifold is equivalent to a linear system or a homogeneous space under some conditions via diffeomorphism [2]. Therefore, it becomes important under which conditions the vector fields on such systems have projections on a homogeneous space.

In this paper, by using differential geometric and Lie theoretic approach, we mention projections of vector fields and then apply them to the control system.

**Keywords**: Lie groups, homogeneous spaces, linear vector fields.
**2010 Mathematics Subject Classification**: 17B66, 16W25, 53C30.

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# On Warped-Twisted Product Submanifolds

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**Abstract**

We investigate warped-twisted product semi-slant submanifolds. We prove that a warped-twisted product semi-slant submanifold of the form ${}_{f_2}M^T \times_{f_1} M^\theta$ with warping function $f_2$ on $M^\theta$ and twisting function $f_1$ is a locally doubly warped product, where $M^T$ is a holomorphic and $M^\theta$ is a slant submanifold of a globally conformal Kaehler manifold. Then, we obtain a general inequality for the squared norm of the second fundamental form of the doubly warped product semi-slant submanifolds and get some results for these types of submanifolds.

**Keywords**: Twisted product, doubly warped product, holomorphic distribution, slant distribution, locally and globally conformal Kaehler manifold.
**2010 Mathematics Subject Classification**: 53C15, 53B20.

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# The quaternionic ruled surfaces in terms of Bishop frame

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**Abstract**

In this paper, we investigate the quaternionic expression of the ruled surfaces drawn by the motion of the Bishop vectors. The distribution parameters, the pitches, and the angle of pitches of the ruled surfaces are calculated as quaternionic.

**Keywords**: Quaternion, spatial quaternion, ruled surface, Bishop Frame, distribution parameter, angle of pitch, the pitch.
**2010 Mathematics Subject Classification**: 11R52, 37E45, 53A04, 53A05.

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# On a new class of Riemannian metrics on the coframe bundle

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**Abstract**

The study of Riemannian metrics in fiber bundles is one of the main problems in the theory of differential-geometric structures on manifolds (see, for example, [1], [2]). This is due to the fact that Riemannian manifolds find applications not only in mathematics, but also in mechanics and physics. In this talk, we introduce a new class of Riemannian metrics in the coframe bundle.

Let $(M, g)$ be an $n-$ dimensional Riemannian manifold and $F^*(M)$ be its coframe bundle [3]. The Riemannian metric $\bar{G}$ is defined on the coframe bundle $F^*(M)$ by the following equalities:

$$\bar{G}({}^H X, {}^H Y) = {}^V(g(X,Y)),$$
$$\bar{G}({}^{V_\alpha}\omega, {}^H Y) = 0,$$
$$\bar{G}({}^{V_\alpha}\omega, {}^{V_\beta}\theta) = 0, \quad \alpha \neq \beta,$$
$$\bar{G}({}^{V_\alpha}\omega, {}^{V_\alpha}\theta) = \lambda(h)g^{-1}(\omega,\theta) + \mu(h)g^{-1}(\omega, X^\alpha)g^{-1}(\theta, X^\alpha)$$

for all vector fields $X, Y$ and 1-forms $\omega, \theta$, where

$$h = \|X^\alpha\|^2 = g^{-1}(X^\alpha, X^\alpha), \lambda(h) = h_\alpha$$

and $\mu(h) = \mu_\alpha$ are some smooth functions such that $\lambda_\alpha > 0$ and $\lambda_\alpha + h\mu_\alpha > 0, {}^H X, {}^{V_\alpha}\omega, {}^V(g(X,Y))$ are denotes the horizontal lift of $X$, the $\alpha$ -th vertical lift of $\omega$ and the vertical lift of $g(X,Y)$ to the coframe bundle $F^*(M)$ , respectively.

The basic properties of the Levi-Civita connection $\bar{\nabla}$ of metric $\bar{G}$ are investigated, and the values of the components $\bar{\Gamma}^K_{IJ}$ of this connection for different indices are also found.

**Keywords**: Coframe bundle, Riemannian metric, horizontal lift, vertical lift, Levi-Civita connection.
**2010 Mathematics Subject Classification**: 53C25, 55R10.

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# On semiconformal curvature tensor in $(k, \mu)$-contact metric manifold

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**Abstract**

The objective of the present paper is to study the $(k, \mu)$-contact metric manifold with the semiconformal curvature tensor. The $(k, \mu)$-contact metric manifold satisfying $P \cdot R = 0$ and semiconformally flat are studied and the conditions under which it is $\eta$-Einstein manifold are established. Further, $P \cdot S = 0$ is investigated and the relation for Ricci tensor is obtained. Also, some results for $\eta$-Einstein $(k, \mu)$-contact metric manifold satisfying the condition $P \cdot S = 0$ are established. Finally, $h$-semiconformally semi-symmetric $(k, \mu)$-contact metric manifold and $\phi$-semiconformally semi-symmetric $(k, \mu)$-contact metric manifold are introduced and shown that non-Sasakian $h$-semiconformally semi-symmetric $(k, \mu)$-contact metric manifold and non-Sasakian $\phi$-semiconformally semi-symmetric $(k, \mu)$-contact metric manifold are $\eta$-Einstein manifold if $\mu \neq 1$ and $\mu \neq \frac{n-1}{n}$ respectively.

**Keywords**: $\eta$-Einstein manifold, $(k, \mu)$-contact metric manifold, $h$-semiconformally semi-symmetric, $\phi$-semiconformally semi-symmetric, semiconformal curvature tensor.
**2010 Mathematics Subject Classification**: 53C25, 53C15, 53D15.

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[3] U.C. De, V.J. Suh,*On Weakly Semiconformally symmetric manifolds*. Acta Math Hungar **157**(2) (2019), 503–521.

# On almost pseudo semiconformally symmetric manifold

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**Abstract**

The object of the present paper is to study a type of Riemannian manifold, namely, an almost pseudo semiconformally symmetric manifold which is denoted by $A(PSCS)_n$. Several geometric properties of such a manifold are studied under certain curvature conditions. Some results on Ricci symmetric $A(PSCS)_n$ and Ricci-recurrent $A(PSCS)_n$ are obtained. Next, we consider the decomposability of $A(PSCS)_n$. Finally, two non-trivial examples of $A(PSCS)_n$ are constructed.

**Keywords**: Pseudo semiconformally symmetric manifold, symmetric manifold, conformal curvature tensor, semiconformal curvature tensor, conharmonic curvature tensor.
**2010 Mathematics Subject Classification**: 53C25, 53C15, 53D15.

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[3] U.C. De, V.J. Suh,*On Weakly Semiconformally symmetric manifolds*. Acta Math Hungar **157**(2) (2019), 503–521.

# Electromagnetism and Maxwell's equations in terms of elliptic biquaternions in relativistic notation

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**Abstract**

In this study, the relativistic transformation equations of electric and magnetic fields which has an important place for Maxwell's equations are investigated by elliptic biquaternions. Firstly, thanks to representations of elliptic Lorentz transformations of elliptic biquaternions are obtained the transformation equations of electric and magnetic fields. After, using the elliptic biquaternionic relativistic transformation relation the obtained the transformation equations of electric and magnetic fields. It is investigated which method is useful by comparing these results.

**Keywords**:Elliptic biquaternions, Maxwell's equations, Electromagnetism
**2010 Mathematics Subject Classification**: 11R52, 83C22, 78A25

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# A Study on Commutative Elliptic Octonion Matrices

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**Abstract**

In this study, firstly notions of similarty and consimilarty are given for commutative elliptic octonion matrices. Then the Kalman-Yakubovich s-conjugate equation is solved for the first conjugate of commutative elliptic octonions. Also, the notions of eigenvalue and eigenvectors are studied for commutative elliptic octonion matrices. In this regard, the fundamental theorem of algebra and Gershgorin's Theorem are proved for commutative elliptic octonion matrices. Finally examples that related to the theorems are given.

**Keywords**: Elliptic octonion matrices, consimilarity, Gershgorin disk.
**2010 Mathematics Subject Classification**: 15B33, 15A18, 13A99, 12A27.

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# Golden Structure on the Cotangent Bundle with Sasaki Type Metrics

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**Abstract**

Golden structure on a Riemannian manifold was constructed using Golden ratio by Crasmereanu and Hretcanu [1, 2]. In [3] Gezer et.al. introduced locally decomposable Golden Riemannian manifold. In this paper, we study papaholomorphy property of the Sasaki metric by using almost paracomplex structure on the cotangent bundle. Then we investigate locally decomposable Golden structure on the cotangent bundle which related to this almost paracomplex structure.

**Keywords**: Cotangent bundle, Sasaki metric, Almost paracomplex structure, Golden structure.
**2010 Mathematics Subject Classification**: 53C07, 53C15.

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# Moving Quaternionic Curves and Modified Korteweg-de Vries Equation

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**Abstract**

In this study, we obtain the modified Korteweg-de Vries (mKdV) equations by the motion of quaternionic curves in 3 and 4-dimensional Euclidean spaces, respectively. For this purpose, the second part of our study is devoted to recalling the basic concepts and related theorems. Then we give the evolutions of quaternionic curves with reference to the Frenet formulae. Finally, we generate the mKdV differential equations with the help of their evolutions.

**Keywords**: Modified Korteweg-de Vries Equation, Quaternionic Curves, Evolution Curves.
**2010 Mathematics Subject Classification**: 53A04, 35Q53.

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# General rotational surfaces in Euclidean spaces

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**Abstract**

The general rotational surfaces in the Euclidean 4-space $R^4$ was first studied by Moore (1919). The Vranceanu surfaces are the special examples of these kind of surfaces. Self-shrinker ows arise as special solution of the mean curvature ow that preserves the shape of the evolving submanifold. In addition, surfaces are the generalization of self-shrinker surfaces. In the present article we consider surfaces in Euclidean spaces. We obtained some results related with rotational surfaces in Euclidean $4-$space $R^4$ to become self-shrinkers. Furthermore, we classify the general rotational surfaces with constant mean curvature. As an application, we give some examples of self-shrinkers and rotational surfaces in $R^4$.

**Keywords**: Mean curvature, self-shrinker, general rotational surface.
**2010 Mathematics Subject Classification**: 14J26, 53A05.

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# Semi-slant Submanifolds of Kenmotsu Manifold with respect to the Schouten-van Kampen Connection

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**Abstract**

Slant immersions in complex goemetry were defined by B. Y. Chen as a natural generalization of both holomorphic immersions and totally real immersions [3]. Examples of slant immersions into complex Eucidean spaces $\mathbb{C}^2$ and $\mathbb{C}^3$ were given by Chen and Tazawa [7] while slant immersions of Kaehler $\mathbb{C}$-spaces into complex projective spaces were given by Maeda, Ohnita and Udagawa [15].

On the other hand [13], A. Lotta has introduced the notion of slant immersion of a Riemannian manifold into an almost contact metric manifold and he has proved some properties of such immersions. Later, R. S. Gupta and et al. studied slant submanifolds of a Kenmotsu manifold [9]. Then, N. Papaghic initiated semi-slant submanifolds. These submanifolds are a generalized version of CR- submanifolds. J. L. Cabrerizo et al. [4] extended the study of semi-slant submanifolds of Kaehler manifold to the setting of Sasakian manifolds. Finally V.A. Khan et al. obtained some basic results pertaining to the geometry of slant and semi-slant submanifolds of a Kenmotsu manifold [25]

In this paper we study integrability of distribution on a semi-slant submanifold of a Kenmotsu manifold with respect to the Schouten-van Kampen connection.

**Keywords**: Slant submanifolds, Semi-slant submanifolds, Kenmotsu manifolds, Schouten-van Kampen connection.
**2010 Mathematics Subject Classification**: 53C15, 53C25, 53A30.

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# Some Special Legendre Mates of Spherical Legendre Curves

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**Abstract**

In this study, we consider some special mates of spherical Legendre curves by using Legendre frame along spherical front or frontal on Euclidean unit sphere. In this sense, we define orthogonal-type and parallel-type spherical Legendre mates. After, we get some characterizations between Legendre curvatures of orthogonal-type Legendre mates. Moreover, we obtain that the evolute and the involute of spherical front correspond to second and third orthogonal-type Legendre mate of spherical front, respectively. Especially, we show that there is no parallel-type Legendre mates of spherical frontal.

**Keywords**: Legendre curve, frontal, front, spherical, Legendre mate, involute, evolute.
**2010 Mathematics Subject Classification**: 53A04, 57R45, 58K05.

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# New results on "fixed-circle problem"

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**Abstract**

In this talk, we give a brief survey of "fixed-circle problem". In this context, we present a new solution to the this problem on a metric space using the number $M(u, v)$ defined as

$$M(u,v) = \max\left\{d(u,v), d(u,Tu), d(v,Tv), \left[\frac{d(u,Tv)+d(v,Tu)}{1+d(u,Tu)+d(v,Tv)}\right] d(u,v)\right\}, \qquad (2)$$

for all $u, v \in X$. Also, we give some illustrative examples to show the validity of the obtained result.

**Acknowledgement:** This work is financially supported by Balıkesir University under the Grant no. BAP 2020 /019.

**Keywords**: Fixed circle problem, fixed circle, fixed disc.
**2010 Mathematics Subject Classification**: 54H25, 47H10, 55M20.

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# New kinds of conformal Riemannian maps

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**Abstract**

In this study, we give definitions of two new kinds of conformal Riemannian maps from an almost Hermitian manifold to a Riemannian manifold, their decompositions and some examples.

**Keywords**: Riemannian maps, conformal Riemannian maps.
**2010 Mathematics Subject Classification**: 53C15, 53C55, 58C25.

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# On framed Tzitzeica curves in Euclidean space

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**Abstract**

Framed curves are frequently used recently to study singular curves, and they have many contributions to singularity theory. In this study, framed Tzitzeica curves are introduced with the help of framed curves. Furthermore, necessary and sufficient conditions are given for some special framed curves such as framed rectifying curves and framed spherical curves to be Tzitzeica curves. Also, differential equations related to these have been created and the solutions of these differential equations based on framed curvatures have been examined. Finally, Tzitzeica equations are given for the general position vector of a framed curve.

**Keywords**: Framed Tzitzeica curves, framed rectifying Tzitzeica curves, framed spherical Tzitzeica curves.
**2010 Mathematics Subject Classification**: 53A04, 53A05, 58K05.

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# On the Projective Equivalence of Rational Algebraic Curves

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**Abstract**

Detecting equivalences of curves is widely studied in the literature, as it directly contributes to fields such as computer vision, computer graphics and pattern recognition. However, it is difficult to find enough studies when the subject is projective equivalences. Recently Hauer and Jüttler published a study [1] in which the projective equivalences of curves were studied comprehensively. In their paper, the authors proposed two methods, which they called the direct method and the reduced method. In this talk, we discuss the answer to the question "Is it possible to solve the problem of detecting projective equivalences of curves with the arguments of differential geometry?". In the light of the results we have obtained, it will be possible to say that the answer to this question is positive, with fast and effective algorithms created by using differential invariants.

**Keywords**: Projective equivalences, projective symmetries, rational curves, differential invariants
**2010 Mathematics Subject Classification**: 14H50, 68W30, 65D18.

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# Locally conformally flat metrics on surfaces of general type

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**Abstract**

We prove a nonexistence theorem for product type manifolds. In particular we show that the 4-manifold $\Sigma_g \times \Sigma_h$ obtained from product of closed surfaces, does not admit any locally conformally flat metric arising from discrete and faithful representations for genus $g \geq 2$ and $h \geq 1$.

**Keywords**: Hyperbolic geometry, locally conformally flat metrics, 4-manifolds
**2010 Mathematics Subject Classification**: 53C20, 53C18, 57M50.

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# On the Regular Maps of Large Genus

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**Abstract**

It is known that the regular maps are topological generalizations of Platonic solids which have been studied by various geometers for thousands of years. The theory of maps and their classification is related to the theory of Riemann surfaces, hyperbolic geometry, and Galois theory. Recently the authors in [1] investigate the regular maps corresponding to the subgroups $\Gamma_0(N)$ of the modular group $\Gamma$. They form regular maps of small genus using the normalizer of $\Gamma_0(N)$ in $PSL(2,\mathbb{R})$. In this presentation we investigate the regular maps of large genus. In order to do that we describe some subgroups of the normalizer which are related to the vertices, edges and faces of the maps. The maps we formed in this way are all regular and of large genus.

**Keywords**: Regular maps, Riemann surfaces, Normalizer
**2010 Mathematics Subject Classification**: 11G32, 05C25, 05C90

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# On curves satisfying the Lorentz Equation in S-manifolds endowed with a particular affine metric connection

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**Abstract**

An $S$-manifold admits a $(1,1)$-type tensor field satisying $\varphi^3+\varphi=0$. The kernel of this map gives a subspace of the tangent space, which is spanned by $s$ characteristic vector fields $\xi_1,...,\xi_s$. In the present study, given an $S$-manifold endowed with a particular affine metric connection, the Lorentz equation is obtained. Using this equation, curves are characterized by their curvature functions, such as geodesics, circles and helices. Finally, an example is given in $\mathbb{R}^{2n+s}(-3s)$.

**Keywords**: $S$-manifold, $\theta_\alpha$-slant curve, metric connection.
**2010 Mathematics Subject Classification**: 53C25, 53C40, 53A04.

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# Some notes on deformed lifts

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**Abstract**

Let now $T^2(M_r)$ be the bundle of 2−jets, i.e. the tangent bundle of order 2 over $C^\infty$−manifold $M_r$, $\dim T^2(M_r) = 3r$ and let

$$(x^i, x^{\bar{i}}, x^{\bar{\bar{i}}}) = (x^i, x^{r+i}, x^{2r+i}), x^i = x^i(t),$$
$$x^{\bar{i}} = \frac{dx^i}{dt}, x^{\bar{\bar{i}}} = \frac{1}{2}\frac{d^2x^i}{dt^2}, t \in \mathbb{R}, i = 1, ..., r$$

be an induced local coordinates in $T^2(M_r)$. It is clear that there exists an affinor field (a tensor field of type $(1,1)$) $\gamma$ in $T^2(M_r)$ which has components of the form

$$\gamma = \begin{pmatrix} 0 & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \end{pmatrix} \tag{3}$$

with respect to the natural frame $\{\partial_i, \partial_{\bar{i}}, \partial_{\bar{\bar{i}}}\} = \{\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^{\bar{i}}}, \frac{\partial}{\partial x^{\bar{\bar{i}}}}\}$, $i = 1, ..., r$, where $I$ denotes the $r \times r$ identity matrix. From here, we have

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ I & 0 & 0 \end{pmatrix}, \quad \gamma^3 = 0, \tag{4}$$

i.e. $T^2(V_r)$ has a natural integrable structure $\Pi = \{I, \gamma, \gamma^2\}$, $I = id_{T^2(M_r)}$, which is an isomorphic representation of the algebra $R(\varepsilon^2)$, $\varepsilon^3 = 0$.

The purpose of this report is to study the deformed lifts (i.e. so called the intermediate and complete lifts) of functions and vector fields which surprisingly appear in the context of algebraic structures in the bundle of 2−jets.

**Keywords**: Holomorphic functions, vertical and complete lift, bundle of 2−jets.
**2010 Mathematics Subject Classification**: 53C07; 53C15.

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# Problems of lifts concerning dual-holomorphic functions

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**Abstract**

We can define the following classical numbers of order two: dual numbers (or parabolic numbers), i.e. $a + \varepsilon b$, $a, b \in \mathbb{R}$, $\varepsilon^2 = 0$, where $\mathbb{R}$ is the field of real numbers. Let $M_n$ be a differentiable manifold and $T(M_n)$ its tangent bundle. Two types of lift (extension) problems have been studied in the previous works: a) The lift of various objects (functions, vector fields, forms, tensor fields, linear connections, etc.) from the base manifold to the tangent bundle; b) The lift on the total manifold $T(M_n)$ by means of a specific geometric structure on $T(M_n)$. In the present report we continue such a study by considering the structure given by the dual numbers on the tangent bundle and defining new lifts of functions, vector fields, forms, tensor fields and linear connections.

**Keywords**: Dual numbers, tangent bundle, complete lift, dual-holomorphic functions, anti-Kähler manifold.
**2010 Mathematics Subject Classification**: 55R10; 57R22; 53C05.

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# On the geometry of $\varphi$-fixed points

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**Abstract**

In this talk, we present some solutions to an open problem related to the geometric study of $\varphi$-fixed points arisen in a recent study. Our methods depend on the usage of appropriate auxiliary numbers such as $M(u,v)$ defined by

$$M(u,v)=\max\left\{d(u,v),d(u,Tu),d(v,Tv),\left[\frac{d(u,Tv)+d(v,Tu)}{1+d(u,Tu)+d(v,Tv)}\right]d(u,v)\right\},$$

for all $u,v\in X$, and

$$\rho=\inf\left\{d\left(Tu,u\right):u\in X,Tu\neq u\right\}.$$

**Acknowledgement:** This work is supported by the Scientific Research Projects Unit of Balıkesir University under the Grant no. BAP 2020/019.

**Keywords**: Fixed point, $\varphi$-fixed point, $\varphi$-fixed disc.
**2020 Mathematics Subject Classification**: 54H25, 47H10.

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# A Survey for Envolute-Involute Partner Curves in Euclidean 3-Space

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**Abstract**

A study on evolute-involute partner curves is observed in Euclidean 3-space. Some theorems and special characterization are given geometrically.

**Keywords**: Evolute, involute, curvature.
**2010 Mathematics Subject Classification**: 53A04, 53A05.

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# A new perspective for the intersection of two ruled surfaces

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**Abstract**

In this study, we give a new approximation for the intersection of two ruled surfaces generated by the natural lift curves by using the isomorphism between the subset of tangent bundle of unit 2-sphere, $T\bar{M}$ and unit dual sphere, $DS^2$. According to Study's map, to each curve on $DS^2$ corresponds a ruled surface in Euclidean 3-space, $IR^3$. Through this correspondence, we have corresponded to each natural lift curves on $T\bar{M}$ unique ruled surfaces in $IR^3$. Exploitting the intersection of these ruled surfaces, we give some significiant properties.

**Keywords**: Ruled surface, tangent bundle, surface intersection.
**2010 Mathematics Subject Classification**: 53A04, 53AO5, 53A17.

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# Hypersurfaces with the lowest center of gravity in space forms

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**Abstract**

Singular minimal hypersurfaces are the critical points of variational integral, so–called $\alpha$–potential energy, and generalize known minimal hypersurfaces. In other words, they minimize potential $\alpha$–energy and create hypersurface models with the lowest center of gravity. Thanks to these properties, they characterize high-dimensional analogues of catenary, which minimizes potential energy under the influence of gravitational force. Therefore, they have great importance in physics and architecture. Historically, the problem of finding singular minimal surfaces stretch away to early studies of Lagrange and Poisson on the equation that models a heavy surface in vertical gravitational field.

In this talk, we address the problem of finding translation hypersurfaces that satisfy the singular minimal hypersurface equation in space forms and express the characterizations for such hypersurfaces.

**Keywords**: Singular minimal hypersurfaces, Translation graphs, $\alpha$–catenary.
**2010 Mathematics Subject Classification**: Firstly 53A10, Secondly 53C42.

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# The Special Curves of Fibonacci and Lucas Curves

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**Abstract**

In this paper, we introduce the contrapedal, radial, inverse, conchoid and strophoid curves of Fibonacci and Lucas curves which are defined by Horadam and Shannon, [1]. Moreover, the graphs of these special curves are drawn by using Mathematica.

**Keywords**: Fibonacci curve, Lucas curve, Contrapedal curve, Radial curve, Inverse curve, Conchoid curve, Strophoid curve
**2010 Mathematics Subject Classification**: 53A04.

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# The charged point-particle trajectories on timelike surfaces

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**Abstract**

The aim of the study is to investigate the variations of the Darboux frame curvatures for curves on the timelike surfaces. The Killing equations in terms of the variations of the Darboux curvatures along the curve is especially derived. Killing equations are used to interpret the motion of the charged point-particles in a magnetic field. The charged particle motion along a curve on a timelike surface is examined through the Killing equations. As an application, the parametric representations of the magnetic curves on the de Sitter space in the Killing magnetic vector field. Also, some motivated examples are presented and visualised through the MAPLE program.

**Keywords**: Applications to physics, Vector fields, Magnetic flows, Ordinary differential equations, Special curves, Variational methods.
**2010 Mathematics Subject Classification**: 53Z04, 53B50, 37C10, 14H45, 14H50, 35A15, 70E17.

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# Rigid motions of the polarization plane in the optical fiber through quaternion algebra

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**Abstract**

In this study, the rigid motion of the polarization plane along the linearly polarized light wave in the optical fiber is investigated. The motion is expressed through the quaternion algebra. Then, the parametric equations of the Rytov curves that are traced curves of the polarization vector are given via quaternion product and matrix form. Moreover, the characterization of the electric field is obtained and the electromagnetic trajectories along the linearly polarized light wave in the optical fiber are obtained by using the variational approach. Finally, the rigid motion of the polarization vector is illustrated and visualized through the MAPLE program.

**Keywords**: Applications to physics, vector fields, magnetic flows, variational methods, quaternion algebras.
**2010 Mathematics Subject Classification**:53Z04, 53B50, 37C10, 11R52, 16H05, 32A07.

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# Some geometric results on $S_b$-metric spaces

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**Abstract**

In this talk, we present some geometric results on $S_b$-metric spaces. To do this, we prove fixed-disc, fixed-ellipse, fixed-hyperbola, fixed-Cassini curve and fixed-Apollonius circle theorems modifying some known contractions. Also, we give some illustrative examples.

**Keywords**: $S_b$-metric space, fixed disc, fixed figure.
**2010 Mathematics Subject Classification**: 54H25, 47H10, 55M20.

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# Chen-Ricci Inequalities for Anti-Invariant Riemanian Submersions From Cosymplectic Space Forms

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**Abstract**

In this talk, we present Chen-Ricci inequalities for anti-invariant Riemannian submersions from Cosymplectic space forms.

**Keywords**: Chen-Ricci inequality, anti-invariant submersion, Cosymplectic space form.
**2010 Mathematics Subject Classification**: 53C40, 53B05, 53A40.

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# A new type of osculating curve in $E^n$

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**Abstract**

In this study, we give a new type of osculating curve in the $n$-dimensional Euclidean space. Using the definition of an osculating curve in lower dimensions we give a generalization of osculating curve in $E^n$. Additionally, we show that the curvatures of the generalized osculating curve construct the solution of a higher order differential equation.

**Keywords**: Osculating curve, unit speed curve, higher order linear differential equation.
**2010 Mathematics Subject Classification**: 53A04, 53A07, 34A05.

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# Generalized Trigonometric B-Spline and Nurbs Curves and Surfaces with Shape Parameters

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**Abstract**

In this study, generalized trigonometric basis (or GT-basis for short) functions along with two shape parameters are formulated and used. The generalized trigonometric B-spline and NURBS curves and surfaces are defined on these basis functions and also analyze their geometric properties which are analogous to classical B-Spline and NURBS curves and surfaces. GT-Spline and NURBS curves meet the conditions required for parametric and geometric continuity. Furthermore, some curve and surface design applications have been discussed for future works.

**Keywords**: GT, B-spline, NURBS.
**2010 Mathematics Subject Classification**: 65D07, 65D17.

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# On $L_1$-pointwise 1-type Gauss map of tubular surface in $\mathbb{G}_3$

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**Abstract**

In this study, we handle a tubular surface whose Gauss map $G$ satisfies the equality $L_1G = f(G + C)$ for the Cheng-Yau operator $L_1$ in Galilean 3-space $\mathbb{G}_3$. We find the tubular surface having $L_1$-harmonic Gauss map and we give an example of this type surface. Moreover, we obtain a complete classification of tubular surface having $L_1$-pointwise 1-type Gauss map of the first kind in $\mathbb{G}_3$ and we give some visualizations of this type surface.

**Keywords**: Cheng-Yau operator, Gauss map, tubular surface.
**2010 Mathematics Subject Classification**: 53A35, 53B30.

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# Some characterizations of spherical indicatrix curves generated by Flc frame

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**Abstract**

In this study, firstly, the vectors of the Flc framework of a curve and the spherical indicator curves drawn by the Darboux vector on the unit sphere surface were defined. Arc length and Frenet vectors were calculated for each spherical indicator curve defined. Last, we have obtained the geodesic curvatures according to both Euclidean space $E^3$ and unit sphere $S^2$ of Flc vectors.

**Keywords**: Polynomial curves, Flc frame, Spherical indicatrix, Geodesic curvature.
**2010 Mathematics Subject Classification**: 53A04, 53A05.

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# On some properties of gradient Ricci-Yamabe solitons on warped product manifolds

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**Abstract**

A Riemannian manifold $(M^n, g)$, $n > 2$ is called a gradient Ricci-Yamabe soliton if

$$Hessf + \alpha Ric = \left(\lambda - \frac{1}{2}\beta scal\right) g,$$

where $f$ is a smooth function on $M$ and $\lambda, \alpha, \beta \in \mathbb{R}$ [1]. We find the main relations for a warped product manifold to be a gradient Ricci-Yamabe soliton. We give some pyhsical applications.

**Keywords**: Ricci-Yamabe flow, Ricci-Yamabe soliton, warped product.
**2010 Mathematics Subject Classification**: 53C21, 53C50, 53C25.

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# Some Estimates in Terms of The Divergencefree Symmetric Tensor and It's Trace

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**Abstract**

In this work, we give some optimal lower bounds for the eigenvalues of the Spin Dirac operator in terms of the Divergencefree Symmetric Tensor and It's Trace. Considering the minimum case eta−Killing spinor is characterized with Killing pair over the Sasakian spin manifolds.

**Keywords**: Spin geometry, Dirac operator, Estimation of eigenvalues.
**2010 Mathematics Subject Classification**: 53C25, 53C27, 34L40

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# On translation-like covering transformations

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**Abstract**

The concept of "translation-like elements" of the group of covering transformations of a covering projection onto a compact space is defined. It is shown that the group of covering transformations of the universal covering projection of a compact Riemannian manifold with negative sectional curvatures admits no non-trivial translation-like elements.

**Keywords**: Covering transformation, negative sectional curvature, translation-like elements.
**2010 Mathematics Subject Classification**: 58A03, 57M10

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# SU(3) Structure on Submanifolds of Locally Conformal Spin(7) Structure with 2-plane Field

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**Abstract**

Let $M$ be an 8-dimensional manifold with the Riemannian metric $g$ and structure group $G \subset SO(8)$. The structure group $G \subset Spin(7)$, then it is called $M$ admits $Spin(7)$-structure. M. Fernandez [1] classifies the all types of 8-dimensional manifolds admitting Spin(7)-structure. In general, torsion-free Spin(7) manifold are studied considerably.

On the other hand, manifolds admitting Spin(7)-structure with torsion have rich geometry as well. Locally conformal parallel structures has bee studied for a long time with Kähler condition is the oldest one. By means of further groups whose holonomy is the exceptional, the choices of the $G_2$ and Spin(7) deserves to attention. Ivanov [3], [4], [5] introduces a condition when 8-dimensional manifold admits locally conformal parallel Spin(7) structure.

Salur and Yalcinkaya [6] studied almost symplectic structure on Spin(7)-manifold with 2-plane field. Fowdar [2] studied Spin(7) metrics from Kähler geometry. In this research, 8-manifold equipped with locally conformal Spin(7)-structure with 2-plane field. Then, almost Hermitian 6-manifold can be classified by the structure of $M$.

**Keywords**: Spin(7) structure, Torsion , Almost Hermitian structure
**2010 Mathematics Subject Classification**: Primary 53D15; Secondary 53C29.

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# A new approach to generalized cantor set for $\mathbb{R}^2$ in fractal geometry

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**Abstract**

Many studies have been done about the fractal geometry from past to present. Especially fractal geometry has been used in many fields such as architecture, art and medicine. Although fractal geometry was first discovered by the French mathematician Benoit Mandelbrot, [1], many fractal like structures were described before his. Cantor set and Sierpinski triangle are examples of fractal defined before Mandelbrot. The most important factor in the increase of fractal geometry studies, especially in recent years, is the developing computer technologies. The Cantor set is one of the most important fractal structures described before Mandelbrot and was described by G.Cantor in 1881,[2]. Afterwards, the length calculation and dimension calculation of this structure were made and iterated function systems were created, [3, 4]. Generalized Cantor set is defined for $[0, 1]$ closed interval, which is a subset of the set of real numbers, [5]. Then, similarly, length and dimension calculations were made for this structure and iterated function systems were created, [6, 7, 8]. In addition, the generalized Cantor Set was defined for the $[a, b]$ interval, [9, 10]. Also, this range was also examined on the curve, [9, 10]. Then similarly, length and dimension calculations were made and iterated function systems were obtained for this fractal structure, [9, 10].

In this study, Cantor set is defined for the $[a, b] \times [c, d]$.Later, when the area calculation was made for this structure the result was found zero. Dimension calculation was made for this structure and that was obtained with iterated function systems. In addition to, this fractal structure was examined on the surface. Finally in this study was completed by giving the cylinder example.

**Keywords**: Fractal geometry, Cantor set, fractal dimension.
**2010 Mathematics Subject Classification**: 28A80, 28A75, 53A05.

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# Some Notes on Ruled Surfaces according to Alternative Moving Frame in Euclidean 3-space

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**Abstract**

In this study, ruled surfaces according to alternative moving frame in Euclidean 3-space are revisited. Their differential geometric properties such as line of striction, distribution parameter, fundamental forms, Gaussian and mean curvatures are obtained in terms of only alternative moving frame apparatus. Moreover, conditions for the base curve and the line of striction to be principal line, asymptotic line, and geodesic curve which are special curves on the surface are investigated. Finally, some related examples are given.

**Keywords**: Alternative moving frame, curvatures, distribution parameter, line of striction, ruled surface.
**2010 Mathematics Subject Classification**: 53A04, 53A05.

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# New Results for Spacelike Bertrand Curves in Minkowski 3-Space

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**Abstract**

One of the most interesting examples of curves theory in Euclidean space is Bertrand curves. Many different studies have been done on these curves for many years and continue to be done. Therefore, we have a great knowledge of the geometric properties of these curves. In [1], the authours gave a new perspective to Bertrand curves. This point of view was also carried to curves in Minkowski 3-space [2, 3]. In this talk, new results for spacelike bertrand curves will be given in the light of recent studies on Bertrand curves.

**Keywords**: Bertrand curves, Minkowski 3-space, spacelike curves.
**2010 Mathematics Subject Classification**: 53C50, 53C40.

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# On the intersection curve of two ruled surfaces in dual space

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**Abstract**

In this study, we examine the conditions of the intersection curve of two ruled surfaces by using E. Study mapping. That is, to each dual curves corresponded ruled surfaces. Then, the intersection of these ruled surfaces is investigated.

**Keywords**: Ruled surface, intersection curve, dual space.
**2010 Mathematics Subject Classification**: 53A04, 53A05, 53A17.

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# Looking at the Concept of Entropy from Information Geometry

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**Abstract**

Entropy has applications in many fields from physics to statistics, from engineering to social sciences in the age of communication and technology in the 21st century., It appears as an application of information theory in areas such as secure data transfer, lossless data compression. The entropy formula created by Shannon using probability calculations is an important measurement method used in information theory [1]. In this study, it is aimed to better understanding the use of entropy concept in information theory with three applications. Some of the uses of Shannon's entropy formula in information theory are shown in this applications.

**Keywords**: Entropy, Information theory, Information gain.

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# New Results for Cartan Null Bertrand Curves in Minkowski 3-Space

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**Abstract**

In this study we consider Cartan null Bertrand curves in Minkowski 3-space. Since the principal normal vector of a Cartan null curve is a spacelike vector, the Bertrand mate curve can be a null curve, a timelike curve and a spacelike curve with spacelike principal normal. The case where the Bertrand mate curve is a null curve, were studied in [1] and proved that a Cartan null curve is a Bertrand curve if and only if it is a null geodesic or a Cartan null curve with constant second curvature. The other cases were studied in [2]. In this talk, new results for Cartan null Bertrand curves will be given in the light of recent studies on Bertrand curves in Minkowski 3-space.

**Keywords**: Bertrand curves, Minkowski 3-space, Cartan null curve, timelike curve, spacelike curve.
**2010 Mathematics Subject Classification**: 53C50, 53C40.

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# Surfaces with a Common Asymptotic Curve in Terms of an Alternative Moving Frame in Lie Group

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**Abstract**

In this paper, we investigate surfaces from a given curve as an asymptotic curve by using the alternative moving frame in a 3-dimensional Lie Group. We get the conditions for that curve being asymptotic. Moreover, we illustrate some examples to support our theory.

**Keywords**: Lie group, Alternative moving frame, Asymptotic curve, Surface family.
**2010 Mathematics Subject Classification**: 53A05, 22E15.

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# On $f$-Biharmonic and $f$-Biminimal Curves in Kenmotsu Manifolds

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**Abstract**

In this study, we obtain some necessary and sufficient conditions for a Frenet curve to be $f$-biharmonic and $f$-biminimal in 3-dimensional Kenmotsu manifolds. We determine these conditions, in detail according to various cases. Besides, we evaluate all the these conditions separately for slant and Legendre curves.

**Keywords**: Kenmotsu Manifolds, $f$-Biharmonic Curves, $f$-Biminimal Curves, Slant Curves.
**2010 Mathematics Subject Classification**: 53C25, 53C43, 58E20.

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# Electromagnetic Curves Through Alternative Moving (N,C,W) Frame

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**Abstract**

In this study, in 3D Riemannian space, the behavior of a linearly-polarized light wave in optical fiber and the rotation of the polarization plane through an alternative moving frame {n,c,w} is investigated. A new geometric phase is modeled in 3D Riemannian space based on the relationship between the geometric evaluation of the polarized light waves and the Berry phase. Thus, polarized plane rotation is defined with the help of Fermi-Walker parallel transport law. Also, a physical interpretation of the results in the optical fiber is presented. Moreover, some examples are visualised through the MAPLE program.

**Keywords**: Applications to physics, Vector fields, Magnetic flows, Ordinary differential equations, Special curves, Variational methods.
**2010 Mathematics Subject Classification**: 53Z04, 53B50, 37C10, 14H45, 14H50, 35A15, 70E17.

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# A New Look on Oresme Numbers: Dual-Generalized Complex Component Extension

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**Abstract**

Our main interest in this paper is to explore dual-generalized complex ($\mathcal{DGC}$) Oresme sequence extension. We investigate special linear recurrence relations and sums statements for $\mathcal{DGC}$ Oresme numbers. Furthermore, we describe recurrence relation of $\mathcal{DGC}$ Oresme numbers in matrix form. We also discuss the theory using doubling approach to $\mathcal{DGC}$ Oresme sequence and investigate all of the notions.

**Keywords**: Oresme sequence, dual-generalized complex number.
**2010 Mathematics Subject Classification**: 11B37, 11B39, 11B83.

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# Special Characterizations for Normal Curves According to Type-2 Quaternionic Frame in $\mathbb{R}^4$

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**Abstract**

Quaternions, used in both theoriticial and applied sciences, were defined by Hamilton in 1843. In the field of differential geometry the characterizations given by Serret-Frenet apparatus are useful and interesting. For this reason the Serret-Frenet formulas for spatial quaternionic and quaternionic curve in $\mathbb{R}^3$ and $\mathbb{R}^4$ were obtained by Bhrathi and Nagaraj [2], respectively. Inspired this work the geometers obtained various quaternionic frames in different spaces [3], [6], [7].

In this study we have focused on a special defined curves that called normal curve. Normal curves in four dimensional space defined as a curve whose position vector fully lies in $\{N_1, N_2, N_3\}$. Focusing on this notion we obtain characterizations for normal curves with respect to type-2 quaternionic frame in $\mathbb{R}^4$.

**Keywords**:Quaternion, normal curve, position vector.
**2010 Mathematics Subject Classification**: First 53a04, Second 53C26.

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# Timelike pythagorean normal surfaces with normal $N = e_3$ in Minkowski space

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**Abstract**

In this study, we investigate timelike pythagorean normal (PN) surfaces with surface normal $N = e_3$ in Minkowski space. We obtain cubic, quintic and quartic timelike PN surfaces by means of bivariate polynomials with split quaternion coefficients. In addition, minimal timelike PN surfaces are also examined. Moreover, we give examples of the cubic, quintic and quartic timelike PN surfaces in the Minkowski space.

**Keywords**: Pythagorean normal surfaces, Minimal surfaces, Minkowski space.
**2010 Mathematics Subject Classification**: 53A35, 53A10.

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# On the Ruled Surfaces of the B-Lift Curves

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**Abstract**

In this study, defined a new curve which is called B-Lift curve and obtained the Frenet vectors of the B-Lift curve. Also, the tangent, normal, and binormal surfaces of the B-Lift curve are introduced. Moreover, the Darboux frame of these ruled surfaces are introduced. Finally, the characterizations of these ruled surfaces are examined.

**Keywords**: B-Lift, Natural Lift, Ruled Surface
**2010 Mathematics Subject Classification**: 53A04, 53A15

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# Contact-Complex Riemannian Submersions and $\eta-$Ricci Solitons

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**Abstract**

In this paper, we study contact-complex Riemannian submersions $\pi : M \to B$ from an almost contact metric manifold $M$ onto an almost complex manifold $B$. Here, we give some necessary conditions for which any fiber of $\pi$ and the base manifold $B$ are $\eta-$Ricci soliton, Ricci soliton, $\eta-$Einstein or Einstein.

**Keywords**: Riemannian manifold, Riemannian submersion, $\eta-$Ricci soliton.
**2010 Mathematics Subject Classification**: 32C25, 53C25.

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# On special submanifolds of the Page space

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**Abstract**

Page manifold is the underlying differentiable manifold of the complex surface, obtained out of the process of blowing up the complex projective plane, only once. This space is decorated with a natural Einstein metric, first studied by D.Page in 1978.

In this talk, we study some classes of submanifolds of codimension one and two in the Page space. These submanifolds are totally geodesic. We also compute their curvature and show that some of them are constant curvature spaces.

Finally, we give information on how the Page space is related to some other metrics on the same underlying smooth manifold.

This talk is based on joint work with R.Sarı. Related paper may be accessed from [6].

**Keywords**: Einstein metrics; Hermitian metrics; Kähler metrics; minimal submanifold; totally geodesic.
**2010 Mathematics Subject Classification**: 53C25, 53C55.

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# Z-tensor on Kenmotsu manifolds

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**Abstract**

In this study, we work on Kenmotsu manifolds admitting $Z$ tensor which is a generalization of Einstein tensor. We consider a special type of quarter-symmetric non-metric connection on a Kenmotsu manifold. Some geometric properties of Kenmotsu manifolds with this connection have been investigated.

**Keywords**: Kenmotsu manifold, quarter-symmetric non-metric connection, $Z-$tensor.
**2010 Mathematics Subject Classification**: 53C05, 53C10, 53C25.

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# Some results on $\alpha-$cosymplectic manifolds

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**Abstract**

In this work, we study on some geometric properties of an $\alpha-$cosymplectic manifold. We examine such manifolds under certain conditions which are related to Ricci curvature tensor and conformal curvature tensor.

**Keywords**: $\alpha-$cosymplectic manifold; conformal curvature tensor; $\eta$-Einstein manifold.
**2010 Mathematics Subject Classification**: 53C15; 53C25; 53D15.

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# $\eta$-Ricci solitons on lightlike hypersurfaces

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**Abstract**

In the present paper, we study the $\eta$-Ricci solitons on the lightlike hypersurfaces of the semi-Riemannian manifolds endowed with a torse-forming vector field. We show some conditions for the lightlike hypersurfaces to be $\eta$-Ricci solitons with the tangential component of the torse-forming vector field on the lightlike hypersurfaces as the potential vector. Besides, we also show the geometric properties of the lightlike hypersurfaces satisfying $\eta$-Ricci solitons and gradient $\eta$-Ricci solitons. In particular, we provide some properties of lightlike hypersurfaces admitting $\eta$-Ricci solitons of $(n+2)$-dimensional semi-Riemannian manifolds of constant curvature endowed with a torse-forming vector field. Furthermore, we investigate the principal curvature of the screen homothetic lightlike hypersurface satisfying $\eta$-Ricci and gradient $\eta$-Ricci solitons.

**Keywords**: Lightlike hypersurface, torse-forming vector field, $\eta$-Ricci soliton, screen homothetic, principal curvature.
**2010 Mathematics Subject Classification**: 53C50, 53C25.

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# Geometry of Kähler manifold endowed with symmetric non-metric connection

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**Abstract**

In the present paper, we study the Kähler manifold endowed with the symmetric non-metric connection. We provide the properties of the symmetric non-metric connection in the Kähler manifold. We find that the 1-form associated with the symmetric non-metric connection is closed if and only if it is also closed with respect to the Levi-Civita connection in the Kähler manifold. Moreover, The complex structure and the Kähler form of the Kähler manifold is parallel with respect to the asymmetric non-metric connection if and only if they satisfy certain conditions. We also find that the Nijenhuis tensor of the Kähler manifold vanishes with respect to the symmetric non-metric connection. Furthermore, we give the curvature tensor, Ricci tensor, Ricci operator, scalar curvature and projective tensor of the Kähler manifold with respect to the symmetric non-metric connection. In addition, we define the Kähler group manifold with respect to the symmetric non-metric connection and show its properties.

**Keywords**: Kähler manifold, Nijenhuis tensor, symmetric non-metric connection, group manifold.
**2010 Mathematics Subject Classification**: 53B15, 53C25, 53C55

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# Conformable Special Curves According to Conformable Frame in Euclidean $3-$Space

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**Abstract**

In this study, the effect of fractional derivatives on curves, whose application area is increasing day by day, was investigated. While investigating this effect, the Conformable fractional derivative of differential geometry, which best suits the algebraic structure, was selected. As a result, many special curves and Frenet frame previously obtained using classical derivatives have been redefined with the help of Conformable fractional derivative.

**Keywords**: Fractional derivative, Conformable Fractional Derivative, Frenet Frame, Special curves, Curvatures.
**2010 Mathematics Subject Classification**: 26A33, 53A04, 53A55

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# Ruled surface generated by constant slope direction vector in Galilean $3-$space

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**Abstract**

In this study, we examine a family of ruled surfaces generated by constant slope vector according to rectifying and normal planes of the base curve in Galilean 3−space. Some important results are obtained with respect to special base curves. Also, examples are given as an application that illustrates the results and graphed.

**Keywords**: Ruled surface, Constant slope, Galilean 3−space.
**2010 Mathematics Subject Classification**: 14H50, 14J26, 53A35.

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# Integral Curves of Special Smarandache Curves

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**Abstract**

In this study, we introduce new adjoint curves which are associated curves in Euclidean space of three dimension. They are generated with the help of integral curves of special Smarandache curves. We attain some connections between Frenet apparatus of these new adjoint curves and main curve. We characterize these curves in which conditions they are general helix and slant helix. Finally, we exemplify them with figures.

**Keywords**:Adjoint curve, helix, slant helix.
**2010 Mathematics Subject Classification**: 53$A$04, 14$H$45, 14$J$50.

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# Spacelike and timelike polynomial helices in the semi-Euclidean space $\mathbb{E}_2^n$

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**Abstract**

In this study, we obtain some families of spacelike and timelike polynomial helices in the $n$-dimensional semi-Euclidean space with index two for $n \geq 4$. These helices have spacelike or timelike or null axes. After, we give some examples of the spacelike and the timelike polynomial helices in the semi-Euclidean space $\mathbb{E}_2^n$ for $n = 4, 5$ and 6.

**Keywords**: Spacelike polynomial helix, timelike polynomial helix.
**2010 Mathematics Subject Classification**: 53A35, 53C50.

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# New results for curve pairs in Euclidean 3-space

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**Abstract**

In Euclidean 3-space, an important subject in the geometry of curves is the study of curve pairs with the help of relations between Frenet vectors of two space curves. In searching for pairs of curves such that the tangents, principal normals or binormals of one may be the tangents, principal normals or binormals of the other, there are six cases to be considered [3]. The well-known curves that arise in these cases are Bertrand, Mannheim and evolutes and involutes curves. Many studies have been done on the geometric properties of these curves and these curves are well known. Two curves, corresponding point for point so that the tangents in corresponding points are parallel, are said to be related by a transformation of Combescure [1, 2, 6]. In this study, such curves were considered and very interesting results were obtained.

**Keywords**:Transformation of Combescure, Bertrand curves, Mannheim curves, helices.
**2010 Mathematics Subject Classification**: 53A04.

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# On $k$-type hyperbolic slant helices in 3-dimensional hyperbolic space

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**Abstract**

In [2], the author considered hyperbolic curves in 3-dimensional hyperbolic space, and construct the hyperbolic frame of the hyperbolic space curves. Also, the author studied the associated curve of a hyperbolic curve in $\mathbb{H}^3$. Hyperbolic curves in $\mathbb{H}^3$ according to their Frenet frame, are characterized in [3].

In this paper, we introduce the notion of $k$-type hyperbolic slant helices in $\mathbb{H}^3$, where $k \in \{0, 1, 2, 3\}$. We give the necessary and sufficient conditions for hyperbolic curves to be $k$-type slant helices in terms of their hyperbolic curvature functions. Finally, we give the related examples.

**Keywords**:$k$-slant helix, hyperbolic curves, 3-dimensional hyperbolic space.
**2010 Mathematics Subject Classification**: 53A04, 53B21.

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# Curves on Lorentzian Manifolds

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**Abstract**

In this study, we introduce Frenet curves in 3-dimensional contact Lorentzian manifolds. We define Frenet equations and the Frenet elements of these curves. We give the curvatures of non-geodesic Frenet curves on 3-dimensional contact Lorentzian Manifolds.

**Keywords**: Contact Lorentzian manifolds, Frenet curves
**2010 Mathematics Subject Classification**: 53D10, 53A04

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# Sesqui Harmonic Curves in LP-Sasakian Manifolds

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**Abstract**

In this study, we examine interpolating sesqui-harmonic spacelike curves in a four-dimensional conformally and quasi-conformally flat and conformally symmetric Lorentzian Para-Sasakian manifold.

**Keywords**: Sesqui-harmonic Map, LP-Sasakian manifolds, Frenet curves
**2010 Mathematics Subject Classification**: 53C25, 53C42, 53C50

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# On the Characterization of a Riemannian map by Hyperelastic Curves

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**Abstract**

In this work, we aim to investigate the characterization of a Riemannian map by means of hyperelastic curves carried by a smooth Riemannian map defined between two Riemannian manifolds. We show that if a horizontal curve on the total manifold is a hyperelastic curve on the base manifold along a Riemannian map, then the Riemannian map is isotropic. In the main theorem, we obtain the geometric properties of the Riemannian map in terms of the notions of umbilical Riemannian maps and the mean curvature vector field in case of a horizontal hyperelastic curve on the total manifold is a hyperelastic curve on the base manifold along the Riemannian map. In addition, we examine the characterization of the Riemannian map under the transport of the classical elastic curve.

**Keywords**: Riemannian map, hyperelastic curve, isotropic Riemannian map.
**2020 Mathematics Subject Classification**: 53B20, 53C42.

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# On the Geometry of a Riemannian Map with Helices

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**Abstract**

We introduce h-isotropic Riemannian map and generalize Ikawa's theorem by using the notion of h-isotropic Riemannian map and characterization of an ordinary helix in a Riemannian manifold. Firstly, we obtain some suplemantary lemmas and theorems for h-isotropic Riemannian maps. Then, we investigate the characterization of the Riemannian map when a horizontal helix on the total manifold is transformed by the Riemannian map to the helix on target manifold.

**Keywords**: h-isotropic Riemannian map, Helix, Umbilical Riemannian map.
**2020 Mathematics Subject Classification**: 53B20, 53C42.

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# On the Bertrand mate of a cubic Bézier curve by using matrix representation in $E^3$

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**Abstract**

In this study we have examined, Bertrand mate of a cubic Bezier curve based on the control points with matrix form in $E^3$. Frenet vector fields and also curvatures of Bertrand mate of the cubic Bezier curve are examined based on the Frenet apparatus of the first cubic Bezier curve in $E^3$.

**Keywords**: Bézier curves, Bertrand mate, cubic Bezier curve
**2010 Mathematics Subject Classification**: 53A04-53A05

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# The area of the Bézier polygonal region contains the Bézier Curve and derivatives in $E^3$

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**Abstract**

We have defined and examined the area of the Bézier polygonal region contains the $n^{th}$ *order* Bézier Curve and its, first, second, and third derivativies based on the control points of $n^{th}$ *order* Bézier Curve in $E^3$. Further the area of the Bézier polygonal region contains the $5^{th}$ *order* Bézier curve and its, first, second, and third based on the control points of $5^{th}$*order* Bézier Curve in $E^3$ are examined too..

**Keywords**: Bézier polygon, $5^{th}$ *order* Bézier Curve, Bézier polygonal region
**Mathematics Subject Classification**: 2010 53A04-53A05

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# Chracterization of PH-Helical curves in Euclidean 4-space

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In differential geometry, the curve theory takes an important place. It is an interesting problem to calculate the speed of a curve on a closed interval whose norm of the velocity vector is equal to square of a polynomial expression. In 1990 the Pythagorean-hodograph curve (PH-curve) was defined by Farouki and Sakkalis in [3]. After 1990, many papers published about PH-curves such as [2, 7, 8]. The method of obtaining a helix from a planar curve given by Izumiya and Takeuchi in [7] was generalized to PH curves by Camcı and İlarslan in [2]. In this study, we get a general method for PH-helical curves in $\mathbb{E}^4$ by using the ideas in [2] and Bradley's equal sums of squares in [1].

**Keywords**:Pythagorean-hodograph Curves, Helical curves, PH-helical curves
**2010 Mathematics Subject Classification**: 65D17, 53A04.

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# Some Soliton Structures on Twisted Product Manifolds

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**Abstract**

In this talk, we focus on certain gradient soliton structures on twisted product manifolds. The theory of Ricci solitons is a fruitful area of study that is often investigated in differential geometry. The relationships between the Einstein-like manifolds and the warped products are well-studied problems. This study extends these problems on the twisted product manifolds and gives more proper relations between the gradient solitons and the Einstein-like manifolds.

**Keywords**: Twisted product, warped product, gradient Ricci soliton, gradient Yamabe soliton.
**2010 Mathematics Subject Classification**: 53C25, 53C40.

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# Slant Submanifolds of Almost Poly-Norden Metric Manifolds

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**Abstract**

In this paper, we introduce the slant submanifolds of an almost poly-Norden metric manifold. We investigate geometric properties of such submanifolds and give some examples.
**Keywords**: Bronze mean, Slant submanifold, Almost poly-Norden manifold.
**2010 Mathematics Subject Classification**: 53C15, 53C40, 53C42.

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# The Darboux Frame of Curves Lying On The Parallel-Like Surfaces in $E^3$

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**Abstract**

In this work, it has been examined curves on parallel-like surfaces in 3-dimensional Euclidean space. First, it's obtained image curve which lying on the parallel-like surface of the parameter curve on the base surface and later calculated Darboux frame for this image curve. Finally, it's compared the geodesic curvature, the normal curvature, the geodesic torsion for these two curves.

**Keywords**: Darboux frame, Parallel-like surfaces, geodesic curvature, normal curvature, geodesic torsion.
**2010 Mathematics Subject Classification**: 53A04, 53A05.

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# On soliton surface associated with nonlinear Schrödinger (NLS) equation

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**Abstract**

The main scope of this presentation is to explain the smoke ring (or vortex filament) equation which can be viewed as a dynamical system on the space curve in $\mathbb{E}^3$. The differential geometric properties the soliton surface associated with Nonlinear Schrödinger (NLS) equation, which is called NLS surface or Hasimoto surface, are investigated by using Darboux frame. Moreover, Gaussian and mean curvature of Hasimoto surface are found in terms of Darboux curvatures $k_n$, $k_g$ and $\tau_g$. Then, we give a different proof of that the $s-$parameter curves of NLS surface are the geodesics of the soliton surface. As applications we examine two NLS surfaces with Darboux Frame.

**Keywords**: Smoke ring equation, Vortex filament equation, NLS surface, Darboux Frame.
**2010 Mathematics Subject Classification**: 14J25 , 53Z05

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# Position vector of spacelike curves by a different approach

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**Abstract**

The purpose of this study is to obtain a characterization of unit speed spacelike curve with constant curvature and torsion in Minkowski 3 - space. According to this purpose, the position vector of a spacelike curve is expressed by a linear combination of its Serret Frenet Frame with differentiable functions. Since a spacelike curve has different kinds of frames, then we investigate the curve with respect to the Lorentzian casual characterizations of the frame. Hence we examine the results in three different cases including different subcases. Moreover, we illustrate some examples for each case.

**Keywords**: Spacelike W- Curves, Constant Curvature, Minkowski Space.
**2010 Mathematics Subject Classification**: 53A04, 53A05

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# Dual Representation of The Ribon Surfaces

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**Abstract**

In this paper, we deal with dual representation of the Ribbon surface. Then, we gave some properties of this surface.

**Keywords**: Ruled surface, Ribbon frame.
**2010 Mathematics Subject Classification**: 53A04, 53A05.

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# $\alpha-$ Sasakian structure on product of a Kähler manifold and an open curve

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Contact geometry is a theoretical subject which has so many applications in the fields of science such as physics and engineering. From thermodynamics to optics, from electrics to motion equations, it has an important place in many areas [1]. Many studies had been carried out on almost contact, contact and Sasakian manifolds which became increasingly important in the 20th century [2]. In addition, the symplectic geometry and Kähler manifolds which have serious applications in many fields are also important topics in mathematics [3]. That's why, it's important to consider contact and comlex manifolds together [6, 7]. In this paper, at first, we study the product manifold of $M = M_1 \times \beta(I)$ where $M_1$ is almost Hermitian manifold with exact 1-form and $\beta : I \to \mathbb{E}^n$ is an open curve. We show that $M$ has a contact structure. After then, by taking $M_1$ as a Kähler manifold with exact 1-form, we establish an $\alpha$-Sasakian structure on $M$ [8, 7].

**Keywords**: Almost Contact Manifold, Contact Manifold, Sasakian Manifold, Product Manifold
**2010 Mathematics Subject Classification**: 53C15,53D10, 53D15

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# The concept of the notion of a figure in two-dimensional Euclidean geometry and its Euclidean invariants

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**Abstract**

In the work, the concept of the notion of a figure (or an image) in two dimensional Euclidean space is introduced. Global G-invariants of two figures for the fundamental groups G of two dimensional Euclidean space are investigated, where G is a group of orthogonal transformations or special orthogonal transformations. The concept of G-equivalence of two figures is given. An application to m-point is also given.

**Keywords**: Figure, invariant, Euclidean geometry
**2010 Mathematics Subject Classification**: 51M04,53A04.

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# Euclidean invariants of plane paths

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**Abstract**

Let $G$ be the group of Euclidean transformations in $R^2$. In this work, global differential invariants of paths are investigated for the group $G$. The $G$-equivalence problem of two paths is solved by using Euclidean invariants. For given two paths with the common differential $G$-invariants, evident forms of all Euclidean transformations that maps one of the paths to the other are found.

**Keywords**: Path, invariant, Euclidean geometry
**2010 Mathematics Subject Classification**: 51M04,53A04.

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# On the intersection curve of implicit hypersurfaces in $\mathbb{E}^n$

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**Abstract**

We present a method for computing the curvatures and the Frenet vectors of the intersection curve of $(n-1)$ transversally intersecting hypersurfaces represented in implicit form.

**Keywords**: implicit curve, intersection curve, transversal intersection, curvatures, Willmore's method.
**2010 Mathematics Subject Classification**: 53A07, 53A04.

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# Position Vectors of Curves in the Isotropic Space $I^3$

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**Abstract**

In this article, we investigate the position vector of an arbitary curve in the isotropic 3-space $I^3$. We obtain the natural representation of the position vector of an arbitrary curve in terms of the curvature and torsion. In addition, we elaborate on some examples and provide their graphs.

**Keywords**: Position vectors, Isotropic space, curves.
**2010 Mathematics Subject Classification**: $53A35$, $53A40$

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# Pedals and primitivoids of frontals in Minkowski plane

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**Abstract**

The many of curves examined by mathematicians are regular curves with nonzero derivatives at all points. Many curves, however, have unusual points known as singular points. It's fascinating to look at the geometric characteristics of curve with singularities [1, 2]. Because it contains a singular point, it is impossible to construct a moving frame of the curve generally. To solve this problem, T. Fukunaga and M. Takahashi introduced the moving frame along frontals and the curvature of Legendre curves [3].

In this study, we look into pedals and primitivoids, both of which have singularities even for regular curves. Especially, we consider frontals, one of the natural singular curves in the Minkowski plane. Then, we define the notions of pedal, anti-pedal and primitive for frontals in the Minkowski plane and examine the relationships between them. Our conclusions are Lorentzian analogue to the results of [6].

**Keywords**: frontal, pedal curve, Minkowski plane.
**2010 Mathematics Subject Classification**: 53A04, 53A05.

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# Parametric Representation of Hypersurfaces Pencil with Common Geodesic in $E_1^4$

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**Abstract**

In this study, we consider the problem of finding a family of hypersurfaces from a given isogeodesic curve. First, we construct the hypersurface family using the linear combination of the Frenet frames of spacelike and timelike curves in Lorentz-Minkowski 4-space. We also obtain necessary and sufficient conditions for spacelike and timelike curves on this family of hypersurfaces to be both parametric and geodesic. Then, we customize these conditions which is obtained with the help of marcing-scale functions. Finally, we give some surface examples to make the presented method understandable and we plot these surfaces by projecting them into 3-dimensional spaces.

**Keywords**: hypersurface, geodesic, Lorentz-Minkowski.
**2010 Mathematics Subject Classification**: 14J70, 53A35.

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# Nearly Cosymplectic Manifolds with Tanaka-Webster Connection

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**Abstract**

The aim of this study is to research concircular curvature tensor of Nearly cosymplectic manifolds with Tanaka-Webster Connection. We defined The concircular curvature tensor with respect to the generalized Tanaka-Webster connection. Also in this work, we studied concircularly flat, $\xi$-concircularly flat, $\phi$-concircularly flat, pseudo-concircularly flat and we have shown some equations.

**Keywords**: Nearly cosymplectic manifolds, Tanaka-Webster connection, concircular curvature tensor
**2010 Mathematics Subject Classification**: 53D10, 53D15, 53C25

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# Certain curves on Riemannian manifolds

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**Abstract**

In this talk, we study certain curves on Riemannian manifolds. We first introduce Bertrand curves on Riemannian manifolds and give a characterization. We also introduce framed curves on Riemannian manifolds and obtain characterization by using vector cross product map.

**Keywords**: Bertrand Curves, Framed Curves.
**2010 Mathematics Subject Classification**: 53C15, 53B20, 53C43.

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# On the ruled surfaces generated by Sannia Frame based on alternative frame

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**Abstract**

In this paper, we define a set of ruled surfaces such that the base curve is taken to be the striction curve of $N$, $C$ and $W$ ruled surfaces and the director curves are the elements of Sannie frame. The characterizations of these surfaces such as fundamental forms and curvatures are also examined to provide the conditions for those to be developable and minimal.

**Keywords**: Ruled surfaces, alternative frame, Sannia frame.
**2010 Mathematics Subject Classification**: 53A04.

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# Characterization of timelike Bertrand curve mate by means of differential equations for position vector

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**Abstract**

The aim of this presentation is to characterize the position vectors of the timelike Bertrand mate in Minkowski space by means of differentiable functions. Therefore, the position vector of a timelike Bertrand curve is obtained by a linear combination of its Frenet frame with differentiable functions. Depending on the curvature and torsion value, different situations that occur for the timelike Bertrand curve are indicated. The relations between the Frenet apparatuses of timelike Bertrand curve mate are obtained.

**Keywords**: Timelike Bertrand curve, Minkowski space, position vectors.
**2010 Mathematics Subject Classification**: 53A35.

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# Characterization of spacelike Bertrand curve mate by using position vector

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**Abstract**

The purpose of this study is to obtain a characterization of spacelike Bertrand curve mate with constant curvature and torsion in Minkowski space. According to this purpose, the position vector of a spacelike Bertrand curve mate is obtained by a linear combination of its Serret Frenet Frame with differentiable functions. Then we investigate the spacelike Bertrand curve mate with respect to the Lorentzian casual characterizations of the frame. Thus we examine the results in threedifferent cases including different subcases

**Keywords**: Spacelike Bertrand curve, Minkowski space, position vectors.
**2010 Mathematics Subject Classification**: 53A35.

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# Biharmonic Curves along Riemannian Submersions and Riemannian Maps

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**Abstract**

In this talk, we study Riemannian submersions and Riemannian maps between Riemannian manifolds. First, we investigate the biharmonicity of the curve along a Riemannian submersion. Then, similarly, we examined the biharmonicity of the curve along a Riemannian map. Moreover, we examined the biharmonicity of the curve along a totally umbilical Riemannian map.

**Keywords**: Riemannian manifold, Riemannian submersion, Riemannian map, biharmonic curve.
**2010 Mathematics Subject Classification**: 53C15, 53B20, 53C43.
This work is supported by TUBITAK.

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# On Tubular Surfaces with Modified Frame in 3-Dimensional Galilean Space

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**Abstract**

In this study, we define modified frame in 3-dimensional Galilean space. Also, we handle helices and tubular surfaces in terms of the modified orthogonal frame in $G_3$ and characterize them.

**Keywords**: Galilean Space, Tubular Surface, Modified Orthogonal Frame.
**2010 Mathematics Subject Classification**: 53A04.

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# Obtaining general terms of polygonal number sequences with areas of unit squares and area formula of right triangle

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**Abstract**

Proof is a way for learning mathematics. In mathematics proof is very important for the statement to enter mathematics literature and to be accepted true by mathematicians. An unproven statement (unless it is an axiom) can not be accepted by mathematicians. The aim of this study is to obtain general terms of polygonal numbers sequences with the help of the areas of unit squares. In this study the areas of unit square and the formula of the right triangle were used to obtain the general terms of polygonal numbers. In this way, the general terms of polygonal numbers were obtained by visualizing them with the help of geometry. It is thought that the method used will make proofs attractive for students who do not like to make mathematical proofs or who are afraid to make mathematical proofs.

**Keywords**: polygonal number, formula of right triangle, area of unit squares .
**2010 Mathematics Subject Classification**: 97G40, 97G70.

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# Graphs with Density

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**Abstract**

In this talk, firstly I will give some basic notations, definitions and theorems about surface with density in Euclidean and Minkowski 3-space. After that, I will give a summary of informations graphical surface in Minkowski 3-space. Moreover, I will consider graphical surface with linear density.Then, I will get the equation of minimal graphical surfaces and characterize some solutions of the equation of minimal graphs in Minkowski 3-space with linear density. Finally, I will give some examples and draw the graphs of minimal surfaces with density for some special cases via Matlab program.

**Keywords**: Surfaces with density, translation surfaces, minimal surface, Minkowski 3-space, graphs, graphical surface

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# Two Different Models for Spatial Boomerang Motion

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**Abstract**

Boomerang motion is a special motion in space that accepts a closed curve as orbit with the same starting and ending points. If the curve is a Bezier curve with the n-control points, the motion becomes controllable form. Control points are the points that make up the Bezier curve. Motion can be handled in two different ways. First of all, starting from the points, the Bezier curve and then the boomerang motion along this curve are defined. The second way, starting from the curve, is the control points, and the curve-based boomerang motion is created. This study will be built on these two different methods.

**Keywords**: Bezier, boomerang, orbit.
**2010 Mathematics Subject Classification**: 53A04, 53A17, 57R25.

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# A note on involute-evolute curves of framed curves in the Euclidean Space

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**Abstract**

The aim of this paper is to introduce involute-evolute curves of framed curves in the Euclidean space. The relationship between the adapted frames of the involute-evolute curve couple and some new characterizations with relation to the curve couple are found.

**Keywords**: Involute, evolute, framed curve.
**2010 Mathematics Subject Classification**: 53A04, 58K05.

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# The geometrical interpretation of the energy in the null cone $\mathbf{Q}^2$

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**Abstract**

In this study, we describe the directional derivatives in accordance with the asymptotic orthonormal frame $\{x, \alpha, y\}$ in $\mathbf{Q}^2$ and we also give the extended Serret-Frenet relations by using cone frenet formulas. Hence, we explain the geometrical understanding of energy on the particle in each asymptotic orthonormal vector fields in null cone. Furthermore, we determine the bending elastic energy function for the same particle in null cone according to curve $x(s, \xi, \eta)$ and we conclude our results by providing energy variation sketches with respect to directional derivatives for different cases.

**Keywords**: Asymptotic orthonormal frame, null cone, energy.
**2010 Mathematics Subject Classification**: 53A40, 53Z05, 35LO5, 37N20.

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# The geodesics on special tubular surfaces generated by darboux frame in $G_3$

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**Abstract**

In this paper, we study geodesics on special tube surfaces generated by the rectifying curves with Darboux frame in Galilean 3-space. In this context, the geodesic formulas are expressed with the help of Clairaut's theorem. Also, we give the Gaussian and mean curvatures of this surfaces.

**Keywords**: Galilean space, tubular surfaces, darboux frame, geodesic curve.
**2010 Mathematics Subject Classification**: 53A35, 53A05, 53C22.

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# Generic Submanifolds of Almost Contact Metric Manifolds

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**Abstract**

Ronsse introduced in [12] the notion of generic and skew CR-submanifolds of almost Hermitian manifolds in order to unify and generalize the notions of holomorphic, totally real, CR, slant, semi-slant and pseudo-slant submanifolds. Other papers, as [18], extended this notion to contact geometry, under the name of almost semi-invariant submanifolds. This class includes the one with the same name introduced by [3], (and studied also in [15]), but without being equal. The class of submanifolds that we introduce and study here in contact geometry, is called by us generic submanifolds, in order to avoid the above confusion, and also since it is different from [18], because in our paper, the Reeb vector field is not necessarily tangent to the submanifold. We obtain necessary and sufficient conditions for the integrability and parallelism of some eigen distributions of a canonical structure on generic submanifolds. Some properties of the Reeb vector field to be Killing and its curves to be geodesics are investigated. Totally geodesic and mixed geodesic results on generic submanifolds are established. We give necessary and sufficient conditions for a generic submanifold to be written locally as a product of the leaves of some eigen distributions. Some examples on both generic submanifolds and skew CR-submanifolds of almost contact metric manifolds are constructed.

**Keywords**: Almost contact metric manifold, Riemannian submanifold, generic submanifold, distribution.
**2010 Mathematics Subject Classification**: 53D10, 53C15.

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# The geometry of a surface in the Riemannian manifold associated with simple harmonic oscillator

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**Abstract**

In this presentation we consider the system of equations for a simple harmonic oscillator as a three-dimensional Riemannian manifold and investigate the intrinsic and the extrinsic geometry of the surface determined by a constant value of a coordinate function. We show that such a surface is not a totally geodesic surface and its intrinsic curvature is strictly negative.

**Keywords**: Simple harmonic oscillator, geometry of a surface, Riemannian geometry.
**2010 Mathematics Subject Classification**: 53B20, 53B25, 34A26

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# Conchoidal Twisted Surfaces in Euclidean 3-Space

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**Abstract**

In this study, we construct the conchoidal twisted surfaces which are generated by synchronized rotations of a planar conchoidal curve in its supporting plane and of this supporting plane about some axis, in Euclidean 3-Space. In addition, we compute the Gaussian and mean curvature of these conchoidal twisted surfaces. Finally, we give the examples of obtained conchoidal twisted surfaces and the graphics of these surfaces are presented in the work.

**Keywords**: conchoidal, surface,twisted, Euclidean space
**2010 Mathematics Subject Classification**: 14J70, 53A35.

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# Study of Isotropic Riemannian Submersions

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**Abstract**

In this paper, we present the notion of isotropic space form submersions between Riemannian manifolds. We first give an example to illustrate this new notion. Then we express a characterization in terms of O'Neill's tensor field $\ddot{T}$ and examine certain relations between sectional curvatures of the total manifold and the base manifold. For an isotropic lift $\dot{M}^n (n \geq 3)$ on a space form $\ddot{M}^{n+p}(c)$ constant sectional curvature $c$, we show that if the mean curvature vector of $\dot{M}^n$ is parallel and the sectional curvature $\ddot{K}$ of $\ddot{M}^n$ satisfies some inequality, then the $\dot{T}$ fundamental tensor of $\ddot{M}^{n+p}$ in $\dot{M}^n$ s parallel and our lift in $\ddot{M}^{n+p}$ is a space form.

**Keywords**: Riemannian submersion, space form, Isotropic submersion, Isotropic immersion.
**2010 Mathematics Subject Classification**: 53C50, 53C25, 53C43

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# Some matrix transformations related to new specified spaces

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**Abstract**

As an introduction, it can be said that one of the non-classical approaches for building new sequence space used recently in summability is that of studying with any infinite matrix. Even though this technique is not easy, it provides a quick technique in obtaining certain results if the inverse of an infinite matrix is present. The main layout of the presentation can be given as follows: In part 1, the kinds of sequence spaces arising in scientific study, including basic concepts, historical developments of some subjects and matrix domain etc are going to be explained. In part 2, the domain $X(\widehat{\widehat{B}})$ within the sequence space $X$ with $X \in \{\ell_\infty, c, c_0, \ell_p\}$ is going to be introduced, and the $\beta-$ and $\gamma-$ duals of $X(\widehat{\widehat{B}})$ will be determined. The Schauder basis of the spaces $c(\widehat{\widehat{B}}), c_0(\widehat{\widehat{B}})$ and $\ell_p(\widehat{\widehat{B}})$ are given after a proof is given about under which conditions the equality $X = X(\widehat{\widehat{B}})$ and inclusion $X \subset X(\widehat{\widehat{B}})$ are valid. In final section, some topological properties of those spaces $c_0(\widehat{\widehat{B}})$, $\ell_1(\widehat{\widehat{B}})$ and $\ell_p(\widehat{\widehat{B}})$ having $p > 1$ are investigated. In part 3, a general theorem which characterizes the matrix transformations from the domain of a triangle matrix into any sequence spaces is stated and also proven. To present the application of this fundamental theorem, a table is given showing the necessary and sufficient conditions for a matrix transformations from $X(\widehat{\widehat{B}})$ to $Y$ in which $X \in \{\ell_\infty, c, c_0, \ell_p\}$ and $Y \in \{\ell_\infty, c, c_0, \ell_p\}$.

**Keywords**: Matrix domain, Schauder basis , $\beta-$ and $\gamma-$duals and matrix transformations.

**2010 Mathematics Subject Classification**: 46A45, 40C05.

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# $T_1$ Limit Spaces

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**Abstract**

The aim of this paper is to characterize each of $T_1$ limit spaces and local $T_1$ limit spaces and to investigate the relationships between them as well as compare $T_1$ limit spaces with the usual one. Morever, we investigate some invariance properties of $T_1$ limit spaces and local $T_1$ limit spaces.

**Keywords**: Topological category, limit spaces, $T_1$ space.
**2010 Mathematics Subject Classification**: 54B30; 18D15; 54A20; 54D10; 54A05.

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# Polygonal Structure Analysis on the Poincare Disk Model

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**Abstract**

In this study, various polygonal structures formed and designed on Poincare disc models were examined. It is known that for regular hyperbolic mosaics represented by the Schlafli symbol $\{p, q\}$ on the Poincare disk model, $p$ gives the number of vertices of the hyperbolic polygons formed on the disk, and $q$ gives the number of branches emerging from any point. Here, while $\{p, q\}$ hyperbolic mosaics are formed, the rule of hyperbolic polygons centered on any point formed by the intersection of mosaic lines has been determined. In $\{p, q\}$ hyperbolic mosaics, starting from the origin point and placing numbers on n layers, triangular structures and numbers on the layers are obtained. In addition, the number of triangular structures formed in the mosaics was evaluated on the layers, and a general form was reached for the number of triangular structures in the intermediate regions of the main triangular regions and the whole mosaic. By placing the corner points of the polygons appearing on the hyperbolic mosaics on the Poincare disk model on the circle layers that we drew, important results were obtained about the number of points in the circle layers and the polygons formed in the rings.

**Keywords**: Hyperbolic geometry, Poincare disk model, Schlafli symbol, regular mosaics.
**2010 Mathematics Subject Classification**: 53A35, 57R60, 30F45, 57M40.

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# Characterizations of a Bertrand Curve According to Darboux Vector

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**Abstract**

In this paper, we first take a Bertrand curve pair and then we use Darboux vector instead of mean curvature vector to give characterizations of Bertrand partner curve by means of the Bertrand curve. By making use of the relations between the Frenet frames of the Bertrand curve pair we give the differential equations and sufficient conditions of harmonicity(biharmonic or 1-type harmonic) of the Bertrand partner curve in terms of the Darboux vector of the Bertrand curve. After driving the conclusions we write an example to demonstrate how our assumptions come true.

**Keywords**: Bertrand, differential equation, biharmonic curve, Darboux, Laplace.
**2010 Mathematics Subject Classification**: 14H45, 53A04.

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# Ricci Solitons on Ricci Pseudosymmetric an Almost Kenmotsu $(\kappa, \mu, v)$-Space

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**Abstract**

Abstract: The object of the present paper is to study some types of Ricci pseudosymmetric an almost Kenmotsu $(\kappa, \mu, v)$-space whose metric tensor admits Ricci soliton. In this case, we investigate the behavior of functions $\kappa, \mu, v$ and $\lambda$. Finally, we characterize the ambient manifold with recpect to these cases.

**Keywords**: Almost Kenmotsu Manifolds, Almost Kenmotsu $(\kappa, \mu, v)$-Space, Ricci Slolitons, Ricci pseudosymmetric, concircular Ricci pseudosymmetric and projective Ricci pseudosymmetric.
**2010 Mathematics Subject Classification**: 53C15, 53C25, 53C08.

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# A Note On the Surfaces in $\mathbb{E}^4$ with Generalized 1-Type Gauss Map

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**Abstract**

Let $M$ be an oriented submanifold of the Euclidean space $\mathbb{E}^n$. $M$ is said to have pointwise 1-type Gauss map if its Gauss map $\nu$ satisfies

$$\Delta\nu = f(\nu + C) \tag{5}$$

for an $f \in C^\infty(M)$ and a constant vector $C$. These kind of surfaces have been studied by many geometers so far (See, for example, [1, 2, 3, 4]). Recently, by describing a condition weaker than (5), Yoon *et. al.* give the definition of generalized 1-type Gauss map in [5, 6]. Namely, $M$ is said to have generalized 1-type Gauss map if $\nu$ satisfies

$$\Delta\nu = f\nu + gC \tag{6}$$

for some $f, g \in C^\infty(M)$ and a constant vector $C$. In this work, we want to announce some new results on surfaces of $\mathbb{E}^4$ with generalized 1-type Gauss map.

**Keywords**: Mean curvature, pointwise 1-type Gauss map, Euclidean spaces
**2010 Mathematics Subject Classification**: 53B25, 53C40

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# Screen Almost Semi-Invariant Lightlike Submanifolds of indefinite Kaehler Manifolds

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**Abstract**

In this study, we introduce screen almost semi-invariant (SASI) lightlike submanifolds of indefinite Keahler manifolds. We obtain a condition for the induced connection to be a metric connection on SASI-lightlike submanifolds and give some characterizations on these manifolds.

**Keywords**: Lightlike Manifolds, Indefinite Kaehler Manifolds, Degenerate Metric, Lightlike submanifolds.
**2010 Mathematics Subject Classification**: 53C15, 53C40, 53C55.

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# Parametric Expressions of Rotational Hypersurfaces According to Curvatures in $E_1^4$

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**Abstract**

In the present study, we obtain the parametric expressions of rotational hypersurfaces according to Gaussian and mean curvatures in $E_1^4$. We find these hypersurfaces when the Gaussian and mean curvatures are constant and zero, seperately. Also, we construct some examples for rotational hypersurfaces with arbitrary curvatures and plot these hypersurfaces' projections into 3-space.

**Keywords**: Rotational hypersurfaces, Mean curvature, Gaussian curvature.
**2010 Mathematics Subject Classification**: 14J70, 53A35.

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# Types and Invariant Parametrizations of Regular and d-Regular Curves

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**Abstract**

Let $\mathbb{R}$ be the field of real numbers and

$$D = \{(a, a*) = a + \varepsilon a * | a, a* \in \mathbb{R}, \varepsilon^2 = 0\}$$

be the algebra of dual numbers. The subset $D_1 = \{(a, a*) : 0 \neq a, a* \in \mathbb{R}\}$ of $D$ is an abelian group with respect to the multiplication operation in the algebra $D$.

For any $A = a + \varepsilon a* \in D_1$ and a transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$, $S(A) = S_A = \begin{pmatrix} a & 0 \\ a* & a \end{pmatrix}$, we define the sets

$\mathbb{D}_1^+ = \left\{ S_A = \begin{pmatrix} a & 0 \\ a* & a \end{pmatrix} | 0 \neq a, a* \in \mathbb{R} \right\}$ and

$\mathbb{D}_1^- = \left\{ \begin{pmatrix} a & 0 \\ a* & a \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} | 0 \neq a, a* \in \mathbb{R} \right\}$. Let us denote $\mathbb{D}_1 = \mathbb{D}_1^+ \cup \mathbb{D}_1^-$. Moreover, we denote the set $\mathfrak{M}\mathbb{D}_1 = \mathfrak{M}\mathbb{D}_1^+ \cup \mathfrak{M}\mathbb{D}_1^-$, where
$\mathfrak{M}\mathbb{D}_1^+ = \{F : \mathbb{R}^2 \to \mathbb{R}^2, F(B) = S_A B + C, A \in D_1, B, C \in \mathbb{R}^2\}$ and
$\mathfrak{M}\mathbb{D}_1^- = \left\{ F : \mathbb{R}^2 \to \mathbb{R}^2, F(B) = (S_A W)B + C, A \in D_1, B, C \in \mathbb{R}^2,\ W = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$.

Let $T = (a, b)$ and $J = (c, d)$ be open intervals of $\mathbb{R}$. A $C^{(2)}-$function $\alpha : T \to \mathbb{R}^2$ for $\forall t \in T$, where $\alpha(t) = (x(t), y(t))$ is called a parametrized curve (path) in the plane. A $T$-path $\alpha(t)$ for $\forall t \in T$ is called regular if $\alpha_1^{'}(t) = x^{'}(t) \neq 0$, $d$-regular if $\alpha_1^{''}(t) = x^{''}(t) \neq 0$. A $T$ -path $\alpha(t)$ and a $J$-path $\beta(t)$ in $\mathbb{R}^2$ are called Dif-equivalent if a $C^{(2)}-$diffeomorphism $\varphi : J \to T$ exists such that $\varphi^{'}(r) > 0$ and $\beta(r) = \alpha(\varphi(r))$ for all $r \in J$. A class of Difequivalent paths in $\mathbb{R}^2$ is called a curve in $\mathbb{R}^2$ . A path $\alpha \in \xi$ is called a parametrization of a curve $\xi$. If a curve $\xi$ has a regular path in it, then it is called a regular curve, if it has a $d$-regular path in it, then it is called a $d$-regular curve. In this study, it is aimed to determine the types of regular and d-regular curves in $\mathbb{R}^2$ and to find the invariant parametrizations with respect to these curve types.

**Keywords**: Dual numbers, parametrik curves (paths), curves, invariant.
**2010 Mathematics Subject Classification**: 53A04, 53A15, 53A55.

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# Dual Quaternions and Translational Surfaces

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**Abstract**

Translational surface is a rational tensor product surface generated from two rational space curves by translating one curve along the other curve. Translational surfaces may also be generated from two rational space curves by dual quaternion multiplication. In this talk, with the aid of dual quaternions, necessary and sufficient conditions are given for a rational tensor product surface to be a translational surface.

**Keywords**: Translational surface, tensor product rational surface, rational space curves, dual quaternion.
**2010 Mathematics Subject Classification**: 14Q10, 14J26, 14H50.

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# Generic $\xi^{\perp}$-Riemannian Submersions from Sasakian Manifolds

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**Abstract**

As a generalization of semi-invariant $\xi^{\perp}$-Riemannian submersions, we introduce the generic $\xi^{\perp}$- Riemannian submersions. We focus on the generic $\xi^{\perp}$-Riemannian submersions for the Sasakian manifolds with examples and investigate the geometry of foliations. Also, necessary and sufficient conditions for the base manifold to be a local product manifold are obtained and new conditions for totally geodesicity are established. Furthermore, curvature properties of distributions for a generic $\xi^{\perp}$-Riemannian submersion from Sasakian space forms are obtained and we prove that if the distributions, which define a generic $\xi^{\perp}$-Riemannian submersion are totally geodesic, then they are Einstein.

**Keywords**: Riemannian submersion, Sasakian manifold, second fundamental form of a map, Einstein manifold.
**2010 Mathematics Subject Classification**: 53C15, 53C25, 53C80.

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# Some remarks on invariant and anti-invariant submanifolds of a golden Riemannian manifold

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**Abstract**

The aim of this paper is to investigate some properties of invariant and anti-invariant submanifolds of a golden Riemannian manifold with the help of induced structures on them by the golden structure of the ambient manifold. Some characterizations of invariant and anti-invariant submanifolds are given, respectively. Also, the totally geodesicity of such types of submanifolds is discussed.

**Keywords**: golden structure, golden Riemannian manifold, invariant submanifold, anti-invariant submanifold.
**2010 Mathematics Subject Classification**: 53C15, 53C25, 53C40.

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# Submersion of CR-Warped Product Submanifold of a Nearly Kaehler Manifold

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**Abstract**

The objective of this article is to study the submersion of $CR$-warped product submanifold of a nearly Kaehler manifold onto an almost Hermitian manifold and it is proved that the holomorphic sectional curvatures of nearly Kaehler manifold and the base manifold coincide. Furthermore, the submersion of mixed totally geodesic $CR$-warped product submanifolds are also studied and several curvature relations are obtained, specifically it is proved that the normal connection of mixed totally geodesic $CR$-warped product submanifold is horizontally flat.

**Keywords**: Riemannian Submersion; Almost Hermitian submersion; $CR$-warped product submanifolds; Nearly Kaehler manifold.
**2010 Mathematics Subject Classification**: 53C15; 53C40; 53C50.

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# Slant Submanifolds of Conformal Sasakian Space Forms

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**Abstract**

Semi-invariant [3] or contact *CR* submanifolds [18], as a generalization of invariant and anti-invariant submanifolds, of almost contact metric manifolds have been studied by a number of geometers. The concept of semi-invariant submanifold was further generalized under name of almost semi-invariant [14]. Several authors studied semi-invariant or contact *CR* submanifolds, and almost semi-invariant submanifolds of different classes of almost contact metric manifolds. Many such references are included in [3], [18], [14], and references cited therein. Since the inception of the theory of slant submanifolds in Kaehler manifolds created by B.-Y. Chen [7], this theory has shown an increasing development. As contact-geometric analogue, there is the concept of slant submanifolds of almost contact metric manifolds [5]. Further, generalizations of slant submanifolds of an almost contact metric manifold are given as a pointwise slant submanifold [11], a semi-slant submanifold [4], a pointwise semi-slant submanifold [11], an anti-slant submanifold [6] (or a pseudo-slant submanifold [2], or a hemi-slant submanifold [10]), a bi-slant submanifold [6], and a quasi hemi-slant submanifold [12]. However, these generalizations turn out to be particular cases of almost semi-invariant submanifolds in the sense of [14], which is contact-geometric analogue of generic submanifold [13] of an almost Hermitian manifold. Tus, either whole or some part of these kind of submanifolds of almost contact metric manifolds is always slant.

The celebrated theory of J.F. Nash of isometric immersion of a Riemannian manifold into a Euclidean space of sufficiently high dimension gives very important and effective motivation to view each Riemannian manifold as a submanifold in a Euclidean space. According to B.-Y. Chen, to establish simple relationship between the main intrinsic invariants and the main extrinsic invariants of a Riemannian submanifold is one of the fundamental problems in the submanifold theory. For a Riemannian submanifold of a Riemannian manifold, the main extrinsic invariant is the squared mean curvature and the main intrinsic invariants include the classical curvature invariants: the Ricci curvature and the scalar curvature. The basic relationships discovered so far are (sharp) inequalities involving intrinsic and extrinsic invariants, and the study of this topic has attracted a lot of attention since the last decade of 20th century. In 1999, B.-Y. Chen ([8, Theorem 4]) obtained a basic inequality involving the Ricci curvature and the squared mean curvature of submanifolds in a real space form. This inequality drew attention of several authors and they established similar inequalities for different kind of submanifolds in ambient manifolds possessing different kind of structures. Motivated by the result of B.-Y. Chen ([8, Theorem 4]), in [9], the authors presented a general theory for a

submanifold of Riemannian manifolds by proving a basic inequality (see [9, Theorem 3.1]), called Chen-Ricci inequality [15], involving the Ricci curvature and the squared mean curvature of the submanifold. Also, in [16], an improved Chen-Ricci inequality was obtained under certain conditions.

The presentation is organized as follows. First, a brief introduction to Sasakian manifolds, Sasakian space forms, conformal Sasakian manifolds, and conformal Sasakian space forms is presented. Next, the concepts of invariant, anti-invariant, semi-invariant, and almost semi-invariant submanifolds of an almost contact metric manifold are presented. It is observed that different kind of slant submanifolds, like invariant, anti-invariant, semi-invariant, $\theta$-slant, pointwise $\theta$-slant, semi-slant, pointwise semi-slant, anti-slant, pseudo-slant, hemi-slant, bi-slant, and quasi hemi-slant submanifolds are particular cases of an almost semi-invariant submanifold of an almost contact metric manifold. Finally, Chen-Ricci inequality involving Ricci curvature and the squared mean curvature of different kind of slant submanifolds of a conformal Sasakian space form tangent to the structure vector field are presented. Equality cases are also discussed.

**Keywords**: Sasakian space form, Slant submanifol
**2010 Mathematics Subject Classification**: 53C25, 53C40

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# Images of Some Discs Under the Linear Fractional Transformation of Special Continued Fractions

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**Abstract**

In this paper, considering that the continued fraction $\mathbf{K}(-1/-k)$ is a Pringsheim fraction, with $n = 1, 2, 3, ...$ for natural numbers $k$ that are $k = 2$ and $k = 3$. The forms of the images of the $\mathbf{K}_n$ discs are examined under the linear fractional transformations $\{S_n\}$ of the complex disc $\overline{\mathbb{D}} = \{w \in \mathbb{C} : |w| \leq 1\}$. Particularly, the relation between Fibonacci numbers and forms of the images of the $\mathbf{K}_n$ are examined for $k = 3$. The results for these special continued fractions from the images of these discs will also be compared with the vertex values on the minimal-length paths in the suborbital graphs. Also, an algorithm is created in Python application language to visually inspect circular disks.

**Keywords**: Continued fractions, Pringsheim, Fibonacci numbers.
**2010 Mathematics Subject Classification**: 11A55, 11B39.

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# Images of Minimal-Length Hyperbolic Paths on the Poincare Disc

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**Abstract**

In this paper, it is aimed to obtain new results and try to create new application areas by using the relations related to the vertex values of the $\mathbf{F}_{u,N}$ suborbital graphs, and the vertex values of the minimal-length paths, which are defined in previous studies. For visual convenience and because the elements of $\Gamma$ sends the hyperbolic lines to hyperbolic lines, we have represented the edges of graphs as hyperbolic geodesics in the upper half plane

$$\mathcal{H} := \{z \in \mathbb{C} \,|\, Im(z) > 0\}$$

that is, as euclidean semi-circles or half-lines perpendicular to $\mathbb{R}$. The vertex values of the minimal-length path obtained in the upper half plane are transferred to the Poincare disc with a special Möbius transform via hyperbolic geometry .

**Keywords**: Suborbital graphs, Poincare, Hyperbolic geometry
**2010 Mathematics Subject Classification**: 20H05, 05C20, 05C05.

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# Almost Yamabe Solitons and Torqued Vector Fields on a Total Manifold of Almost Hermitian Submersions

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**Abstract**

The aim of the present paper is to introduce almost Hermitian submersion from an almost Hermitian manifold admitting almost Yamabe solitons and torqued vector fields. We mainly focus on Kaehler submersions from Kaehler manifolds which are special case of almost Hermitian submersions. We obtain the scalar curvatures of any fibre of such submersion and the base manifold and give the characterization for the soliton. Moreover, We get necessary and sufficient conditions for the total manifold of an almost Hermitian submersion is an almost Yamabe soliton with recurrent, concurrent and torqued vector fields, respectively.

**Keywords**: Almost Yamabe soliton, Torquet vector field, Kaehler manifold.
**2010 Mathematics Subject Classification**: 53C25, 53C43, 53C50.

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